Original Article

Investigation on ultrasonic vibration effects on plastic flow behavior of pure titanium: Constitutive modeling

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A B S T R A C T

Ultrasonic vibration (UV) assisted plastic forming technology is an ideal way to improve the formability of titanium and its alloys at room temperature, due to the softening effect which is more effective than thermal input. Thus, UV assisted compression test of commercially pure titanium is carried out and the deformation behavior is focused in this study. For an accurate prediction of the flow behavior and further application in UV forming process, the original Johnson–Cook (JC) model and its modified formulae have been adopted to construct the constitutive relation of pure titanium deformed with high-frequency oscillation. The yield stress reduction is related to both the amplitude and its square. The other material constants in JC model are established as the function of the amplitude. The comparative analysis of the predicted stress through the built JC model with the measured one is performed, showing the improved JC model can precisely describe the stress-strain relation in a large deformation range at different vibration amplitudes and strain rates.

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1. Introduction

Since the ultrasonic vibration (UV) induced “softening effect” was reported by Blaha and Lengenecker [1] in tensile test of zinc single crystal, UV has been widely applied in metal forming, such as forging [2], extrusion [3], deep drawing [4], equal channel angular pressing [5] and incremental forming [6]. In these processes, UV is expected to lessen the forming force [7], improve the formability [8], reduce the friction between the specimen and the tools [9,10], and thus raise the level of surface quality [11]. The flow resistance reduction during metal shaping when superimposing UV was assumed by the acoustic energy absorption preferentially by lattice imperfections, such as dislocations or grain boundaries [8,12], causing localized heating in these regions and resulting in lower activation energy and better dislocation mobility. Based on the EBSD analysis, UV was found to cause the grain refinement, crystallite rotation and then weaken the Goss texture in the compression aluminum sample [13].

In the UV assisted plastic forming process for high-performance parts, designing the suitable mold geometry and optimizing the process conditions are critical, but always achieved by high-cost trial-and-error. As a cost-effective alternative, numerical modeling and simulation of UV assisted shaping have gained much attention [14–16], of which the forming behavior prediction accuracy is fully dependent on the effectiveness of the constitutive model constructed by a relatively small quantity of UV tensile/compression tests. Therefore, researchers have proposed many material models based on the physical aspect of material behavior.

On the basis of the UV assisted upsetting results in aluminum, Yao et al. [17] proposed a physical-based acoustic plasticity model by modeling the acoustic softening and acoustic residual hardening effects through the thermal activation theory and dislocation evolution theory, respectively. To distinguish the stress superposition and acoustic softening mechanisms in the stress reduction when exciting the UV in the uniaxial tension of copper foil [18], the acoustic softening was modeled using Yao’s method and a power-law relation between the stress superposition and stress reduction was assumed according to the experimental data. The acoustic softening of Al 6063 alloy subjected to transverse UVs during uniaxial tensile tests was modeled through the dislocation density theory based on average parameter [19], which correlated very well with the experimental observation. Based on the mechanical threshold stress theory, Shi et al. [16] proposed a constitutive equation including the acoustic softening effect and validated in the application of the friction stir welding of Al 6061-T6 plates. In the ultrasonic-assisted compression tests of pure aluminum A1100 [20], three different ultrasonic volume effects such as stress superposition, acoustic softening and dynamic impact, were observed and analyzed. At meanwhile, a hybrid model for stress superposition is established, taking into account the elastic deformation of the experimental apparatus and thus fitting well with the experimental results. Sedaghat et al. [15] investigated the UV induced acoustic stress and its effect on material softening behavior, as well as constructed a physics-based constitutive relation by combining acoustoelasticity, dislocation dynamics, acoustic stress transmission, and thermal activation mechanism. This new model was verified by the comparison of predictions with the experiments of upsetting, press forming, and incremental forming of Al 1050.

Though many physical-based models had been developed, they are always complex since they require precisely controlled experiments to obtain more data. Moreover these models need a larger number of material constants than phenomenological models which may not be readily available in the open literature, and thus not always preferable [21,22].

As a typical empirical model, Johnson–Cook (JC) model and its modified forms had been broadly developed and conveniently integrated into kinds of commercial finite element analysis software. These type models are very simple but can describe the material constitutive relations with adequate accuracy and reliability using a limited number of experimental data and material constants [23–25]. Both JC model and its modified versions had been used to describe the tensile behaviors of typical high-strength alloy steel and the determination method for each parameter had been detailed in Ref. [23]. The hot compression behaviors of the homogenized 6026 aluminum alloy under a wide range of deformation temperatures and strain rates had been represented by the original and modified JC models [25]. The modified JC model often provided an accurate and precise estimate of the flow stress for the studied metals, since it considered not only the yield and strain hardening phenomenon, but also the coupled effects of the strain, strain rate and temperature. Up to date, the suitability of JC model and its modified versions in UV assisted forming process have not yet been studied.

The literature review shows that UV has been largely superimposed in the forming process of low-strength metals like Al and Cu, and the effect of the UV on the flow-stress-reduction was quantitatively investigated by complicated physics-based constitutive model, while its application in high-strength material is still scarce [8]. It is noted that UV is a good cure for shaping high-strength metallic materials of low formability and limited ductility at room temperature, like titanium [11] and its alloys [26–28], which has been wildly used in aerospace, marine and medicine due to their high strength-to-weight ratio and corrosion resistance. UV can level up the formability and the workpiece’s quality with a higher softening efficiency than thermal input [29]. However, the effect mechanism of UV in titanium forming process has been scarcely revealed and any constitutive modeling has not come into view.

Therefore, in the present study, UV assisted compression tests on pure titanium are conducted to analyze the effect of the UV amplitude on the forming behavior during compression process, and establish the constitutive relation including UV parameters using the preferable JC model and its modifications.

2. Experimental procedures

The material investigated in this study is the pure titanium TA1, which is firstly machined into column specimens with both diameter and height of 4 mm. To remove the residual work hardening, the specimens are annealed at 600 °C for 2 h and cooled in the furnace filled with argon, resulting in equiaxial structure with average gran size of 22 μm. To ensure a smooth contact with compression platens, two end surfaces of each specimen are polished by fine sandpaper.

UV assisted compression experiments are carried out on the SANS CMT4204 universal testing machine with a maximum loading of 20 kN, as shown in Fig. 1.

UV device is installed on the machine upper crosshead and provides a longitudinal vibration with a frequency f of 28 kHz by a piezoelectric ceramic transducer. The vibration is excited along to the compression direction and amplified by the ultrasonic horn and the specimen, transmitting the ultrasonic energy to the specimen. The ultrasonic amplitude a applied on the specimen is of 3 μm, 4.32 μm, 5.77 μm and 6.93 μm, respectively. Therefore, the sound energy density outputted by 40Cr horn can be computed according to $E = \rho a^2 (2\pi f)^2$, as 2200/J/m³, 4563/J/m³, 8140/J/m³ and 11,742/J/m³, respectively.

All compression tests are implemented at room temperature. The movable crosshead moves up to compress the specimen with a constant stroke of 2 mm, i.e. a total engineering strain of 0.5. To investigate the compression speed
dependency, the crosshead speed is set as 0.96 mm/min, 2.4 mm/min, 24 mm/min and 240 mm/min, corresponding to the initial strain rate $\dot{\varepsilon}$ of 0.004 s$^{-1}$, 0.01 s$^{-1}$, 0.1 s$^{-1}$ and 1 s$^{-1}$, respectively. To reduce the surface/antifriction effect of the high-frequency vibration on the constitutive relation, both surfaces of specimens are lubricated with polytetrafluoroethylene (PTFE). Each test is repeated three times to assure the reliability of the experimental data.

### 3. Results and discussions

#### 3.1. Effect of the ultrasonic vibration on flow stress

To investigate the thermal effect induced by the UV on the material behavior, a handheld thermal infrared imager (Fluke Ti110) is employed to measure the temperature change of the specimen. The thermal image of the sample after UV assisted compression test is shown in Fig. 1 and it can be seen that the temperature only grows from 31.1 °C to 40.8 °C. The temperature incensement may result from the deformation of the sample and the absorption of the UV energy. However, this small variation of 9.7 °C affects slightly on the material deformation behavior and thus the thermal softening caused by UV can be ignored, which is similar to the phenomenon discovered in Refs. [8,30].

To comprehensively study the coupling effects of the press speed and UV amplitude on the material performances, 4 different strain rates and 4 different UV amplitudes are considered in the UV compression experiments. The stress–strain curves under conventional and UV assisted compression tests are depicted in Fig. 2.

From the above figures, it can be seen that the flow stress increases with the augment of the strain and strain rate, which is corresponding to the work hardening phenomenon caused by the rapid increase of dislocation density. In addition, the high-frequency oscillation is applied when the compression load reaches 1200 N, avoiding the slip out of specimen from the work platen if the vibration excites at the beginning. One can easily observe that the flow stress falls immediately when superimposing the vibration near 100 MPa loading. Both the acoustic softening and stress superposition are the main effect mechanisms of UV on the flow stress reduction [30]. However, these two mechanisms are not facely distinguished due to the low sample frequency of the employed load cell, which cannot catch the load oscillation caused by stress superposition [18]. To quantitatively see the influence of the vibration amplitude and the strain rate on the flow stress, the initial yield stress under different conditions are listed in Table 1.

From Table 1, the yield stress increases when the strain rate goes up for a given amplitude and decreases with the increase of the vibration amplitude for a given strain rate. The work hardening and UV softening (including acoustic softening and stress superposition) will affect the mechanical properties of material in turn. When the larger amplitude is applied, UV softening takes the dominance and the flow stress declines. The largest reduction of the yield stress is 66.4 MPa when the vibration amplitude is 6.93 mm and the strain rate is 0.004 s$^{-1}$, which is 25.4 % of the yield stress. A further analysis considering the initial yield stress variation when one of the influence factors (vibration amplitude, strain rate) is fixed and the other one varies, and the results are presented in Fig. 3.

In view of the above graphs, the yield stress varies to almost the same level when fixing the amplitude or strain rate and varying the other one. It implies that the mutually independent action of the UV softening and work hardening by the strain rate. Although the platen moves with oscillation, its average speed does not change as it keeps in contact with the work specimen.

#### 4. Constitutive model

#### 4.1. Johnson–Cook model

The stress–strain data from the compression tests with/without UV can be used to construct the constitutive equations. Due to its simple form, the Johnson–Cook model stands out among the empirical and semi-empirical models. It can effectively express the stress–strain relation in large ranges of strain, strain rates and temperatures, with numbers of successful applications for different metallic materials.

In the present study, compression tests are performed at room temperature and no evident temperature rise is
detected. Therefore the thermal effect is negligible, and the original JC model can be described as,

$$\sigma = (A + B \varepsilon^n) \left(1 + C \ln \dot{\varepsilon}^* \right)$$  \hspace{1cm} (1)$$

where $\sigma$ is the equivalent flow stress, $\varepsilon$ is the equivalent plastic strain, $A$ is the yield stress at reference strain-rate, indicating the stress resistance to micro plastic deformation, $B$ is the coefficient of strain-hardening, $n$ is the strain-hardening exponent, describing the strain hardening behavior. $C$ is the coefficient of strain-rate hardening, which reflects the sensitivity of material to strain rate. $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_0$ is the dimensionless strain rate and $\dot{\varepsilon}_0$ is the reference strain rate.

It can be found that the original JC model requires 4 material constants $A$, $B$, $C$ and $n$, which can be determined by few experiments. To obtain the material constants in conventional and UV assisted compression tests, the reference strain rate $\dot{\varepsilon}_0$ is taken as the minimum value of $0.004 \text{s}^{-1}$. In the following, the evaluation method of the material constants for conventional compression test will be demonstrated.

Fig. 2 – The experimental stress–strain curves at amplitude of (a) 0 µm, (b) 3 µm, (c) 4.32 µm, (d) 5.77 µm, (e) 6.93 µm.
4.1.1. JC constitutive model without vibration

A is the yield stress at the reference strain rate and can be measured as 261.8 MPa. At the reference strain rate, JC model can be expressed as

$$\sigma = A + B\varepsilon^n$$  \hspace{1cm} (2)

Taking natural logarithm on both sides and rearranging Eq. (2), gets

$$\ln(\sigma - A) = n\ln\varepsilon + \ln B$$ \hspace{1cm} (3)

Using the stress–strain data at the reference strain rate, the relationship $\ln(\sigma - A)$ vs. $\ln\varepsilon$ and a fitting line are depicted in Fig. 4(a). $B$ and $n$ can be evaluated as the intercept and the slope of the fitting line, 955.1 MPa and 0.826, respectively.

To determine the material constant $C$, the stress–strain results obtained at different strain rates are required. The original Johnson–Cook model can be rearranged as

$$\frac{\sigma}{A + B\varepsilon^n} = C\ln\dot{\varepsilon}^* + 1$$ \hspace{1cm} (4)

By selecting a series of plastic strain ranging from 0.05 to 0.3, the relation between $\sigma/(A + B\varepsilon^n)$ and $\ln\dot{\varepsilon}^*$ can be calculated and plotted in Fig. 4(b). $C$ can be computed as 0.00986 which

![Fig. 3 – Stress variation compared at different (a) amplitudes, (b) strain rates.](image)

![Fig. 4 – Relation between (a) $\ln(\sigma - A)$ and $\ln\varepsilon$, (b) $\sigma/(A + B\varepsilon^n)$ and $\ln\dot{\varepsilon}^*$ in the original JC model.](image)
is the slope of the fitting line. Therefore the original JC model for pure titanium without oscillation can be shown as

\[ \sigma = \frac{261.8 + 995.1 \cdot \dot{\varepsilon}^{0.826}}{1 + 0.00986 \ln \left( \frac{\dot{\varepsilon}}{0.004} \right)} \]  

(5)

4.1.2. JC constitutive model with vibration

Repeating the above procedures, the material constants in JC model for TA1 compressed with different vibration amplitudes can be calculated and listed in the following table (Table 2).

To clearly show the influence of the amplitude on the material constants, the material constants without and with oscillation are distinguished and noted using the subscript ‘0’ and ‘a’, respectively. The variation of \( A_0/A_a \), \( B_0/B_a \), \( n_0/n_a \) and \( C_0/C_a \) with amplitude \( a \) can be computed and plotted in Fig. 5.

Fig. 5(a) presents the variation tendency of \( A_0/A_a \). It can be found that the ratio increases with the augment of the vibration amplitude. Many researches regarding the volume effect of UV on material behaviors have revealed that the flow stress reduction amount is dependent on the ultrasonic energy density \( E = \alpha \dot{\varepsilon}^2 (2\pi f)^3 \). Huang et al. [31] found a linear relationship between the yield stress reduction and ultrasonic energy. However, Langenecker [12] and Siddiq and Sayed [29] proposed that the stress reduction by UV is proportional to the square root of the ultrasonic energy. Namely, the stress reduction is proportional to the square of amplitude \( a^2 \) in the former opinion and the stress decline is proportional to the amplitude \( a \) in the latter. In the present study, according to the trend showing in Fig. 5(a), the yield stress ratio is assumed as a function of both \( a \) and \( a^2 \).

From Fig. 5(b), the ratio \( B_0/B_a \) is always closed to 1 when the amplitude changes. The maximum and minimum values are 1.025 and 0.964 respectively, with a variation less than 3.6%. Therefore the impact of the vibration on the strain hardening coefficient can be neglected and thus supposing \( B_0/B_a = 1 \). From Fig. 5(c), \( n_0 \) grows with the increase of the amplitude, showing a quadratic relation to \( a \). The variation of \( C \) is indicated in Fig. 5(d), and a linear relationship between \( C \) and \( a \) can be assumed.

From the above, the relation between the material constants and the amplitude can be concluded as

\[
\begin{align*}
A_0/A_a &= k_1 a^2 + l_1 a + 1 \\
B_0/B_a &= 1 \\
n_0/n_a &= k_2 a^2 + l_2 a + 1 \\
C_0/C_a &= k_3 a + 1
\end{align*}
\]  

(6)

where \( k_1, k_2, k_3, l_1 \) and \( l_2 \) are the parameters to be determined. The fitting curves are plotted in red color in Fig. 5 and the parameters can be calculated as 0.0067, 0.00111, 0.118, 0.000286, 0.

Finally, the relation among the stress, the strain, the vibration amplitude and the strain rate is constructed as

\[
\sigma = \frac{261.8 + 995.1 \cdot \dot{\varepsilon}^{0.826}}{1 + 0.00986 \ln \left( \frac{\dot{\varepsilon}}{0.004} \right)} \]  

(7)

4.1.3. Analysis of the original JC constitutive equation accuracy

The flow stress predicted by the established JC model is drawn in Fig. 6, compared with the experimental flow stress.

From the above figures, one can find that when the strain ranges in \((0, 0.2)\), the predicted stress agrees well with the experimental one only at the reference strain rate of \(0.004 \text{ s}^{-1}\). At the other strain rates, the flow stress is underestimated and the derivation increase with the growth of the strain rate. After the strain exceeds 0.25, the predicted stress becomes larger than the experimental value, and the accordance is no longer kept. The maximum relative error between the predicted flow stress and measured stress is 10.8% when the strain, the strain rate and the amplitude are 0.3, \(1 \text{ s}^{-1}\) and 5.77 \(\mu\text{m}\), respectively. In conclusion, the great deviation in prediction implies that the original JC model cannot adequately represent the flow behaviors of the pure titanium in UV assisted compression.

4.2. Modified Johnson–Cook model

To improve the accuracy of the constitutive equation defined by Johnson–Cook model for typical high-strength alloy steel, Lin et al. [23] modified the original JC model by adding a quadratic term of strain and the coupling effect of the temperature and strain rate on the flow behaviors. The modified JC model proposed by Lin et al. [23] can be expressed as

\[
\sigma = \left( A + B_1 \dot{\varepsilon} + B_2 \dot{\varepsilon}^2 \right) \left( 1 + C_1 \ln \dot{\varepsilon}^* \exp \left[ \left( \lambda_1 + \lambda_2 \ln \dot{\varepsilon}^* \right) \left( T - T_0 \right) \right] \right)
\]  

(8)

where \( B_1 \) and \( B_2 \) are the strain-hardening coefficients with respect to the linear and quadratic terms of the strain respectively. \( C_1 \) is strain rate hardening factor. The third term presents coupling effect of the temperature and strain rate, which can be neglected at room temperature. \( A \) is always the initial yield at different conditions, which is the same as the one in the original JC model.

4.2.1. Constitutive model without vibration

At the reference strain rate, Eq. (7) can be simplified to

\[
\sigma = A + B_1 \dot{\varepsilon} + B_2 \dot{\varepsilon}^2
\]  

(9)

By fitting the experimental stress–strain curve at reference strain rate shown in Fig. 7, the values of \( A, B_1 \) and \( B_2 \) can be estimated as 261.8 MPa, 1781.55 MPa and \(-2282.68 \text{ MPa}\).

\( C_1 \) can be determined by rearranging Eq. (7) like

\[
\frac{\sigma}{A + B_1 \dot{\varepsilon} + B_2 \dot{\varepsilon}^2} = C_1 \ln \dot{\varepsilon}^* + 1
\]  

(10)

Using the stress–strain data measured at different strain rate, the \( \sigma/(A + B_1 \dot{\varepsilon} + B_2 \dot{\varepsilon}^2) \) vs. \( \ln \dot{\varepsilon}^* \) curve can be obtained and thus the material constant \( C_1 \) can be calculated as 0.0113 by linear fitting method.

4.2.2. Constitutive model with vibration

Using the determination way demonstrated in Section 4.2.1, the material constants in the modified JC model for different vibration conditions can be evaluated and listed in Table 3.

The effects of the amplitude on \( B_{10}/B_{20}, B_{20}/B_{20} \) and \( C_{10}/C_{20} \) are depicted in Fig. 8.
Fig. 5 – Influence of the amplitude on the parameter ratio (a) \( A_0/A_v \), (b) \( B_0/B_2 \), (c) \( n_0/n_1 \), (d) \( C_0/C_1 \) in the original JC model.

<table>
<thead>
<tr>
<th>Amplitude/μm</th>
<th>A/MPa</th>
<th>B_1/MPa</th>
<th>B_2/MPa</th>
<th>C_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>246.8</td>
<td>1722.83</td>
<td>-2170.81</td>
<td>0.01365</td>
</tr>
<tr>
<td>4.32</td>
<td>228.5</td>
<td>1708.64</td>
<td>-2138.22</td>
<td>0.01490</td>
</tr>
<tr>
<td>5.77</td>
<td>211.1</td>
<td>1666.50</td>
<td>-2058.17</td>
<td>0.01584</td>
</tr>
<tr>
<td>6.93</td>
<td>195.4</td>
<td>1646.05</td>
<td>-2017.20</td>
<td>0.01718</td>
</tr>
</tbody>
</table>

From the above figure, it can be easily seen that \( B_{10}/B_{1v} \), \( B_{20}/B_{2v} \) and \( C_{1v}/C_{10} \) grows linearly with increasing the vibration amplitude. Therefore the following relation can be assumed,

\[
\begin{align*}
B_{10}/B_{1v} &= k_4 a + 1 \\
B_{20}/B_{2v} &= k_5 a + 1 \\
C_{1v}/C_{10} &= k_6 a + 1
\end{align*}
\]

(11)

where \( k_4, k_5 \) and \( k_6 \) are parameters, of which the values can be determined as 0.0115, 0.0182, 0.0720 from the fitting curves in red color, as shown in Fig. 8.

Above all, the constitutive equation of the pure titanium under UV assisted compression can be built using the modified Johnson–Cook model as

\[
\sigma = \left[ \frac{261.8}{0.0067 \dot{\varepsilon}^2 + 0.00286 \dot{\varepsilon} + 1} + \frac{1781.55}{0.0115 \dot{\varepsilon} + 1} - \frac{2282.68}{0.0182 \dot{\varepsilon} + 1} \right]^2 \\
\left[ (0.000814 \dot{\varepsilon} + 0.0113) \ln \left( \frac{\dot{\varepsilon}}{0.004} \right) + 1 \right]
\]

(12)

4.2.3. Analysis of constitutive equation accuracy

The comparison between the measured flow stress and the predicted one by the modified Johnson–Cook model is presented in Fig. 9.

One can find in the above figures that the prediction is in good accordance with the experimental data when the material is deformed in strain range \( (0, 0.4) \) at the strain rate of 0.004 s\(^{-1}\), with a maximum relative error of 1.42%. At the strain rate of 0.1 s\(^{-1}\) and 1 s\(^{-1}\), the stress is undervalued when the strain ranges in \( (0, 0.1) \), but overestimated as the strain larger than 0.3. The relative error reaches 5.85%. Furthermore, the difference between the predicted stress and the measured one keeps enlarging when the strain higher than 0.4. Therefore, the modified Johnson–Cook model cannot give an accurate and
precise estimate of flow behavior of the pure titanium in large deformation.

4.3. Improved Johnson–Cook model

Through the Johnson–Cook models in original form and modified by Lin et al. [23], the acquired constitutive equations of the studied material compressed with oscillation is not able to describe the flow behavior when the strain larger than 0.25 and 0.4, respectively. To give a better prediction of material performance, JC model is further improved with addition of a cubic power term of the strain, according to the stress tendency, expressed as

\[
\sigma = \left[ A + B_3 \varepsilon + B_4 \varepsilon^2 + B_5 \varepsilon^3 \right] \left( 1 + C_2 \ln \dot{\varepsilon}^* \right)
\]  

(13)
Fig. 7 – Relation between (a) $\sigma$ and $\varepsilon$, (b) $\sigma/(A_1 \varepsilon + B_2 \varepsilon^2)$ and $\ln \dot{\varepsilon}^*$ in the modified JC model.

Fig. 8 – Influence of the amplitude on the parameter ratio (a) $B_{10}/B_{1v}$, (b) $B_{20}/B_{2v}$, (c) $C_{10}/C_{1v}$ in the modified JC model.
4.3.1. Constitutive model without vibration

Also setting the reference strain rate as 0.004 s\(^{-1}\), the improved JC model can be rewritten as,

\[ \sigma = \sigma_0 + C_2 \dot{\varepsilon} + B_3 \varepsilon + B_4 \varepsilon^2 + B_5 \varepsilon^3 \]

where \( B_3 \), \( B_4 \) and \( B_5 \) are the strain-hardening coefficients accompanying with the linear, quadratic and cubic terms of the strain respectively. \( C_2 \) is strain rate hardening factor.

Fig. 9 – Comparison of the flow stress–strain curves predicted by the modified JC model with the measured ones at amplitude of (a) 0 \( \mu \)m, (b) 3 \( \mu \)m, (c) 4.32 \( \mu \)m, (d) 5.77 \( \mu \)m, (e) 6.93 \( \mu \)m.
Table 4 – Material constants in the improved JC model at different amplitudes.

<table>
<thead>
<tr>
<th>Amplitude/μm</th>
<th>B₀/MPa</th>
<th>B₄/MPa</th>
<th>B₆/MPa</th>
<th>C₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1823.42</td>
<td>−3144.54</td>
<td>1879.66</td>
<td>0.01177</td>
</tr>
<tr>
<td>4.32</td>
<td>1813.84</td>
<td>−3067.27</td>
<td>1785.25</td>
<td>0.01385</td>
</tr>
<tr>
<td>5.77</td>
<td>1750.62</td>
<td>−2943.04</td>
<td>1706.16</td>
<td>0.01500</td>
</tr>
<tr>
<td>6.93</td>
<td>1743.75</td>
<td>−2875.77</td>
<td>1624.22</td>
<td>0.01609</td>
</tr>
</tbody>
</table>

Table 5 – Comparisons of the measured flow stress with one predicted by the improved JC model.

<table>
<thead>
<tr>
<th>a/μm</th>
<th>ε</th>
<th>i/s⁻¹</th>
<th>Predicted/MPa</th>
<th>Measured/MPa</th>
<th>Relative error/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>−</td>
<td>0</td>
<td>0.004</td>
<td>261.80</td>
<td>261.8</td>
<td>0</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>264.28</td>
<td>272.3</td>
<td>3.77</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>280.73</td>
<td>293.7</td>
<td>−4.42</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>1</td>
<td>591.60</td>
<td>598.9</td>
<td>−1.22</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>605.54</td>
<td>608.2</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>619.48</td>
<td>620.7</td>
<td>−0.20</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0</td>
<td>646.72</td>
<td>652.7</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>622.56</td>
<td>621.0</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0</td>
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σ = A + Bᵢε + B₄ε² + B₆ε³  

By fitting the stress-strain curve at reference rate shown in Fig. 10(a), B₃, B₄ and B₅ can be evaluated as 1907.33 MPa, −3369.41 MPa and 2048.34 MPa. Then rearranging Eq. (12) to the following form

σ = A + B₃ε + B₄ε² + B₆ε³ = 1 + C₂ ln i⁺  

Plotting the graph presented in Fig. 10(b), C₂ can be determined as the slope which is 0.0101.

4.3.2. Constitutive model with vibration
The material constants of the improved JC model for describing the material behavior under UV assisted compression can be evaluated and presented in Table 4. The relation among the constants and amplitude is plotted in Fig. 11.

In Fig. 11, B₃₀/B₃₉, B₄₀/B₄₉, B₅₀/B₅₉ and C₂₀/C₂₉ linearly increase with the growth of the amplitude, and the relation can be defined as

\[ \begin{align*}
B₃₀/B₃₉ &= k₇a + 1 \\
B₄₀/B₄₉ &= k₈a + 1 \\
B₅₀/B₅₉ &= k₉a + 1 \\
C₂₀/C₂₉ &= k₁₀a + 1
\end{align*} \]  

where the parameters k₇, k₈, k₉ and k₁₀ can be acquired from Fig. 11 as 0.0140, 0.0245, 0.0355, 0.0819. Therefore,
Fig. 10 – Relation between (a) $\sigma$ and $\epsilon$, (b) $\sigma/(A + B_3\epsilon + B_4\epsilon^2 + B_5\epsilon^3)$ and $\ln \dot{\varepsilon}^*$ in the improved JC model.

Fig. 11 – Influence of the amplitude on the parameter ratio (a) $B_{30}/B_{3v}$, (b) $B_{40}/B_{4v}$, (c) $B_{50}/B_{5v}$, (d) $C_{20}/C_{2v}$ in the improved JC model.
the improved Johnson–Cook model expressing the material behavior in vibration assisted compression is established as

\[
\sigma = \left[ \frac{261.8}{0.0067a^2 + 0.00286a + 1} + \frac{1907.33}{0.014a + 1} - \frac{3369.41}{0.0245a + 1} \right]^2 \\
+ \left[ 0.0355a + 1 \right] \left[ 1 + (0.00083a + 0.0101) \ln \dot{\varepsilon} \right] 
\] (17)

4.3.3. Analysis of constitutive equation accuracy

The stress–strain relation predicted at different vibration amplitudes and strain rates using the improved JC model is presented in Fig. 12, as well as the measured results.

To clearly see the precision of the built constitutive equation, the relative error between the predicted stress and measured one is listed in Table 5.

Fig. 12 – Comparison of the flow stress–strain curves predicted by the improved JC model with the measured ones at amplitude of (a) 0 µm, (b) 3 µm, (c) 4.32 µm, (d) 5.77 µm, (e) 6.93 µm.
It can be clearly observed that the maximum relative error in the flow stress estimate is only −4.48\%. Also, the improved Johnson–Cook model can give more accurate estimate of the flow behavior in a larger strain range (0, 0.65) than the original and modified JC models. The results demonstrate that the improved Johnson–Cook model proposed in this study is adequate to express the compression behavior of the pure titanium with or without the UV.

4.4. Verification of the constitutive equations

In order to validate the developed constitutive models, a new compression test at different strain rates was performed by superimposing the UV with frequency of 28 kHz and amplitude of 4.81 μm. The predicted stress–strain curves from the above-mentioned three JC models are compared to the newly measured stress–strain data, which is presented in Fig. 13.

From the above figure, one can obviously observe that the experimental stress quickly enlarges when the plastic strain is less than 0.2. In this stage, the original, modified and improved JC models are almost similar and all can well reflect the real deformation behavior. With increasing the deformation, the original JC model severely overestimates the stress due to only one term of the strain with an exponent of near 0.8. The linear and quadratic terms of the strain in the modified and improved JC models makes them quite match the measured ones. The continuous augment of the strain led to an underestimation of the real flow stress. However, the improved JC model agrees well with the measured stresses up to a large strain value of 0.7, indicating a good prediction of the deformation behavior of pure titanium under UV assisted forming process. Hence, the developed constitutive equation of the pure titanium is applicable, and it could be used to obtain reliable simulation results.

5. Conclusions

Ultrasonic vibration (UV) assisted compression behavior of pure titanium is studied in this paper to establish the constitutive relation, for a good forming load and quality prediction when using UV in the cold shaping of titanium with high strength and low formability. The constitutive equation of pure titanium under compression with vibration is constructed using the original JC model, the modified JC model proposed by Lin et al. [23] and the proposed JC model in this study. The following conclusions can be drawn based on this study.

(1) UV can significantly decrease the yield stress with a reduction of 25.4\%, when the vibration amplitude is 6.93 μm. The
work hardening by strain rate and softening by UV affect simultaneously the mechanical properties. UV softening dominates as a larger amplitude is applied.

2. The reduction of yield stress has a relationship to both the ultrasonic amplitude and its square. The strain hardening exponent $n$ is proportional to the square of the amplitude. Other material constants are linear to the vibration amplitude according to the relation graphs.

3. Comparing to the measured stress, the stress predicted by the proposed JC model has a good accordance, with the maximum absolute relative error of 4.48% in a large strain range (0, 0.65) at different vibration amplitudes and strain rates. That indicates the proposed JC model is very suitable for the flow behavior prediction of pure titanium with and without UV.

**Conflicts of interest**

The authors declare no conflicts of interest.

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**References**


