Constitutive analysis and dynamic recrystallization behavior of as-cast 40CrNiMo alloy steel during isothermal compression

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Abstract

Casting-forging combination forming process is an advanced manufacturing technique which is applicable to manufacture the parts with both complex shape and high performance. As a fundamental research of casting-forging combination forming process, as-cast 40CrNiMo alloy steel is obtained through vacuum casting by using metal mold. The isothermal compression tests of as-cast 40CrNiMo alloy steel are implemented on a Gleeble-3800 thermal simulation machine at deformation temperatures of 800, 900, 1000 and 1100 °C, with strain rates of 0.001, 0.01, 0.1, 1 and 10 s⁻¹. The results indicate that the true stress-strain curves present typical dynamic recovery type under low deformation temperature and high strain rate. With the increases of deformation temperature or the decreases of strain rate, the true stress-strain curves gradually transform to dynamic recrystallization type. The Arrhenius-type constitutive equation with Zener–Hollomon parameter is determined for constitutive analysis. The kinetic model and kinematic model of dynamic recrystallization are deduced to describe the dynamic recrystallization behavior.

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1. Introduction

The casting-forging combination forming process is a combination of casting and forging, which has the technological characteristics of both casting and forging. So it has the advantages of simplifying production process and improving material utilization. This manufacturing technology is applicable to manufacture the parts with both complex shape and high performance, the parts can be firstly formed to a near final shape by casting, and then formed to the final shape by forging. The microstructure of parts can be transformed from casting structure to forging structure through the dynamic recrystallization mechanism [1–3].

In the development of the casting-forging combination forming process, Kim et al. [4] proposed tie-rod ends can be manufactured by casting-forging process, an optimal config-
uration of the cast preform was determined. Wang et al. [5] suggested the casting-forging process can be applied in the manufacturing of large aluminum flange, it can reduce press capacity and material costs. A casting-forging forming process of alternator poles was investigated by Chen et al. [6] and Song et al. [7] Their reports indicated that casting-forging forming process can not only shorten process, decrease material and power demand, but also can increase the forming accuracy and mechanical properties.

A comprehensive analysis of microstructure and mechanical properties of casting-forging, low-pressure-casting and rheo-casting of A356 aluminum alloys was investigated by Lee et al. [8]. The results proved that tensile properties and fracture toughness of the cast-forged alloy are more superior. They also researched the effects of eutectic silicon particles on mechanical properties of A356 aluminum alloys fabricated by casting-forging, low-pressure-casting and squeeze-casting. The cast-forged alloy also has the best mechanical properties [9].

In order to enhance the mechanical properties of Mg alloy components, a double control forming technology combining die casting and forging was proposed by Jiang et al. [10–13]. The microstructure and mechanical properties of A356 aluminum alloy wheels prepared by thixo-forging combined with a low superheat casting process were explored by Wang et al. [14]. Results indicated that the tensile strength and elongation of thixo-forged wheels are higher. The casting-forging combination forming technology was suggested to manufacture automobile brake bracket by Zhou et al. [15], and the process parameters and mechanical properties are researched. It is observed that the studies of casting-forging combination forming process are focused on the process optimization and mechanical property analysis. The reports of the characteristics of as-cast material are very few.

40CrNiMo (AISI 4340) alloy steel is a medium carbon low alloy steel. Due to a good balance of strength, toughness and wear resistance, it is widely used to manufacture automotive components, power transmission gears, shaft, and structural parts [16,17]. There have been some research reports of hot deformation and dynamic recrystallization behaviors of 40CrNiMo alloy steel. Hot deformation characteristics and dynamic recrystallization behavior of 4340 steel were investigated by Sajadifar et al. [18,19]. Hot compression tests were performed at a temperature range of 900–1200 °C, a strain rate range of 0.01–1 s⁻¹, and a strain of 0.9. The results proved that the evolution of dynamic recrystallization (DRX) grain structures can be accompanied by a considerable migration of grain boundaries. The activation energy obtained in their research is 427.2 kJ/mol. Sanrutsadakorn et al. [20] researched the initiation of dynamic recrystallization in AISI 4340 steel. The results showed the DRX occurred during hot deformation started when the normalized critical stress and strain reached the values of 0.735 and 0.324, respectively.

The deformation behavior of AISI 4340 alloy steel under high strain rates was investigated by Lee et al. [21]. The tests were performed at strain rates ranging from 500 to 3300 s⁻¹ and constant temperatures ranging from 25 to 1100 °C on a split Hopkinson bar. The results indicated that the flow stress of AISI 4340 alloy steel increases with the increase of strain rate, but decreases with the augmentation of temperature. The adiabatic shear failure mode predominates in the fracture behaviour. The features of dislocations and the precipitation of particles are changed in accordance with the variation of the strain rates and the loading temperatures.

The processing maps of 4340 steel were constructed by Aneta et al. [22] for optimization of the hot forging parameters. The isothermal compression tests are performed at the temperatures ranging from 800 to 1200 °C and at the strain rates in the range of 0.01–100 s⁻¹. It was found that the temperature range of 1050–1200 °C and strain rate range of 3–57 s⁻¹ shows the best parameters of processing, and these conditions can lead to the occurrence of dynamic recrystallization.

However, it should be pointed out that all the above studies are about as-rolled 40CrNiMo alloy steel. These research results are not applicable to the process analysis of casting-forging combination forming. Therefore, the hot deformation and dynamic recrystallization behaviors of as-cast 40CrNiMo alloy steel are investigated. The Arrhenius-type constitutive equation with Zener–Hollomon parameter is determined for constitutive analysis. The kinetic model and kinematic model of dynamic recrystallization are deduced to describe the dynamic recrystallization behavior. The research results of this study can provide basis for the process analysis and numerical simulation of casting-forging combination forming in 40CrNiMo alloy steel.

2. Materials and methods

As-cast 40CrNiMo alloy steel is obtained by vacuum casting using vacuum induction melting furnace and metal mold with the process parameters casting temperature 1550 °C, pre-heating temperature of metal mold 300 °C, casting time 5 s. Cooling method is furnace cooling. The size of casting blank is 42 mm in diameter and 235 mm in height. The chemical components of casting blank are detected by using optical emission spectroscopy method (quantometry analysis). The result (wt%) is 0.415C–0.64Si–0.78Mn–0.74Cr–0.62Ni–0.206Mo–0.0085S–0.021P–(bal.)Fe.

Specimens for isothermal compression are cut from casting blank at different positions using wire cut electrical discharge machining. Specimen size is 10 mm in diameter and 15 mm in height. The cutting positions are shown in Fig. 1. The three positions separately located at the surface zone, transition zone and core zone. The metallographic structures of the three positions are shown in Fig. 2. They are all typical casting dendritic microstructure. The average grain size of each position is tested three times in different areas by using composite grid method according to the ASTM standard. The average grain size is the average value of the three test results. Position 1 is 240.03 μm, Position 2 is 256.45 μm, and Position 3 is 268.94 μm. The average grain sizes of the three positions has little difference, which indicate that the microstructure of the casting blank is relative uniform.

The isothermal compression tests are implemented on a Gleeble-3800 thermal simulation machine at deformation temperatures of 800, 900, 1000 and 1100 °C, with strain rates of 0.001, 0.01, 0.1, and 10 s⁻¹. These are 16 test samples for each position, a total of 48 test samples. Each surface of the specimen is covered with the high temperature lubricant and
tantalum foil to minimize friction. The flow chart of isothermal compression is shown in Fig. 3. The specimen is initially heated to 1200 °C at 10 °C/s (step 1) and held for 120 s for complete austenitizing (step 2). Then, the specimen is cooled to deformation temperature at 10 °C/s (step 3) and held for 180 s for temperature homogenization (step 4). After that, the specimen is compressed under a constant strain rate (step 5), reduction rate in the height is 60%. After the compression deformation, the specimen is quickly quenched into cold water to keep the microstructure (step 6). The quenched specimen is sliced along the axial section, and then polished and etched with saturation picric for the observation of the prior austenite grain boundaries.

3. Results and discussion

3.1. Hot deformation behavior

The true stress-strain curves of the three positions at deformation temperature of 900 °C and 1100 °C are shown in Fig. 4. In the figure, Position 1 with solid line, Position 2 with dash line, Position 3 with dash & dot line. The results show that the hot deformation behavior of the three positions is very similar. It also proves that the as-cast structure of this casting blank is relatively uniform. The true stress-strain curves of the three positions all show that the true stress increases with the increase of the strain rate, and it decreases with the increase of deformation temperature. The true stress-strain curves of the three positions present the same type and variation trend under the same deformation conditions, especially under the deformation condition of high deformation temperature and low strain rate. Therefore, the influence of initial grain size for this batch as-cast 40CrNiMo can be negligible.

The follow-up study is carried out by using the isothermal compression tests data of Position 1. Fig. 5 shows the true stress-strain curves of Position 1 under different deformation conditions. The true stress-strain curves present dynamic recovery type under low deformation temperature and high strain rate. With the increases of deformation temperature or the decreases of strain rate, the true stress-strain curves gradually transform to dynamic recrystallization type that the stress-strain curves present unimodal pattern.

3.2. Constitutive analysis

The Arrhenius-type constitutive equation is adopted for constitutive analysis. In addition, the influence of the forming temperature and the strain rate on the hot deformation behavior can be described by Zener–Hollomon parameter Z [23,24], as shown in Eqs. (1)–(3).

\[ \dot{\varepsilon} = A\sigma^n \exp\left(-\frac{Q}{RT}\right) \]  

\[ F(\sigma) = \begin{cases} \sigma^n & \sigma \sigma < 0.8 \\ \exp(\dot{\varepsilon}) & \sigma > 1.2 \\ [\sin h(\sigma)]^n & \text{for all } \sigma \end{cases} \]  

Fig. 1 – Cutting positions.

Fig. 2 – Metallographic structure of casting blank.

Fig. 3 – Flow chart of isothermal compression.
Fig. 4 – True stress-strain curves of the three positions.

\[ Z = \dot{i} \exp \left( \frac{Q}{RT} \right) \]  \hspace{1cm} (3)

Where \( \dot{i} \) is the strain rate (s\(^{-1}\)), \( \sigma \) is the flow stress (MPa), \( Q \) is the activation energy (J·mol\(^{-1}\)), \( R \) is the gas constant (8.314 J·mol\(^{-1}\) K\(^{-1}\)), \( T \) is the absolute temperature, and \( A, n_1, \alpha, n, \beta (\beta = \alpha n_1) \) are the material constants. The formula of flow stress can be derived as follow.

\[ \sigma = \frac{1}{\alpha} \ln \left\{ \left( \frac{Z}{A} \right)^{1/n} + \left( \frac{Z}{A} \right)^{2/n} + 1 \right\}^{1/2} \]  \hspace{1cm} (4)

The peak stress, maximum stress in the true stress-strain curve, is usually used to construct the constitutive equation.

Since that is meaningful to predict the maximum load in hot-working process. The values of peak stress under different deformation conditions are listed in Table 1. Base on the peak stress, the material constants of constitutive equation are constructed through linear regression method. The average slope of the linear fitting curves of \([\ln \sigma - \ln \dot{i}]\) under different deformation temperatures is accepted as \( n_1 \). The average slope of the linear fitting curves of \([\ln \sigma - \ln \dot{i}]\) under different deformation temperatures is accepted as \( \beta \), and then \( \alpha \) can be solved by \( \alpha = \beta / n_1 \). The average slope of the linear fitting curves of \([\ln(\sinh(\omega \sigma)) - \ln \dot{i}]\) is accepted as \( n \). The average slope of the linear fitting curves of \([1000/T - \ln(\sinh(\omega \sigma))]\) under different strain rates is accepted as \( Q/(RN) \), then \( Q \) (kJ·mol\(^{-1}\)) can
be solved. At last, the intercept of the linear fitting curve of $\ln[\sinh(\alpha \dot{\varepsilon})] - \ln Z$ is accepted as $\ln A$, then $A$ can be solved. The linear fitting curves for each material constant are shown in Fig. 6(a)–(e). The values of the material constants are: $n_1 = 7.172$, $\dot{\varepsilon} = 6.981 \times 10^{-2}$, $\alpha = 9.733 \times 10^{-3}$, $\dot{\varepsilon} = 5.147$, $Q = 332.1 \text{ kJ/mol}$, and $A = 5.544 \times 10^{12}$. The activation energy of as-casting 40CrNiMo is 332.1 kJ/mol, that is less than as-rolled 40CrNiMo (427.2 kJ/mol) [18,19].

The Arrhenius-type constitutive equation of as-cast 40CrNiMoB alloy steel is shown in Eq. (5). Fig. 6(f) shows the linear fitting curve of the measured value from isothermal compression tests versus the predicted value from constitutive equation. The $R$ square is 0.992, which reveals the predicted data calculated by Eq. (5) has a high accuracy.

\[\sigma = \frac{1}{9.733 \times 10^{-3}} \ln \left\{ \left( \frac{Z}{5.544 \times 10^{12}} \right)^{1/5.147} + 1 \right\}^{1/2} \]

\[Z = \dot{\varepsilon} \exp \left( \frac{3.321 \times 10^{5}}{8.314T} \right) \]  

**Table 1 – Values of peak stress under different deformation conditions (MPa).**

<table>
<thead>
<tr>
<th>Strain rate ($\text{s}^{-1}$)</th>
<th>800</th>
<th>900</th>
<th>1000</th>
<th>1100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>100.15</td>
<td>62.044</td>
<td>40.241</td>
<td>24.687</td>
</tr>
<tr>
<td>0.01</td>
<td>145.30</td>
<td>89.793</td>
<td>59.652</td>
<td>41.071</td>
</tr>
<tr>
<td>0.1</td>
<td>199.95</td>
<td>131.65</td>
<td>86.124</td>
<td>53.08</td>
</tr>
<tr>
<td>1</td>
<td>234.97</td>
<td>170.97</td>
<td>123.63</td>
<td>90.426</td>
</tr>
<tr>
<td>10</td>
<td>257.82</td>
<td>209.36</td>
<td>160.52</td>
<td>124.24</td>
</tr>
</tbody>
</table>

**Fig. 6 – Linear fitting curves for constitutive equation.**
3.3. Dynamic recrystallization behavior

During the hot-working process, the effects of work hardening and dynamic softening are concomitant. In initial stage of metal hot-working process, the proliferation and interaction of dislocation cause the work hardening, and the effect of work hardening is much greater than dynamic softening, the true stress presents linear growth. As the hot deformation progresses, the accumulated energy of the material increases continuously. Driven by the accumulated energy and thermal activation energy, the effect of dynamic softening (dynamic recovery softening) is enhanced by the combination and recombination of dislocations. The increasing rate of true stress slows down. After the stress reaches the critical value (critical stress \( \sigma_c \)), if the dislocation density under a low level, the driving force for the nucleation of dynamic recrystallization will be insufficient. In this case, the true stress will remain basically unchanged after reaching the peak value (peak stress \( \sigma_p \)). This means that only dynamic recovery occurred, and the true stress-strain curve presents the dynamic recovery type. Rather, if the dislocation density is high enough for the nucleation of dynamic recrystallization, the dynamic recrystallization mechanism will be activated. In general, the effect of the dynamic softening (mainly dynamic recrystallization softening) can overcome the effect of the work hardening. In this case, the true stress will drop suddenly after reaching the peak value, and the true stress-strain curve presents the dynamic recrystallization type [25,26], as shown in the Fig. 7.

As shown in Fig. 5, when the deformation temperatures at 900°C–1100°C, with strain rates of 0.001 s\(^{-1}\)–0.1 s\(^{-1}\), the true stress-strain curves present the dynamic recrystallization type. Fig. 8 shows the microstructures of as-cast 40CrNiMo alloy steel after isothermal compression. These are some banded shadows in the metallographic photos which are derived from the dendrite segregation in the casting process of alloy steel. The dendrite segregation is concentrated in the interdendritic regions and elongated during the compression process. The dendrite segregation can be eliminated by subsequent heat treatment.

Fig. 8(a) and (b) shows that recrystallized grains have formed, but there are still some coarse casting grains. Because of the recrystallization is not sufficient in these deformation conditions. Partial initial casting structure is replaced by forging structure through incomplete dynamic recrystallization.

It is different from Fig. 8(a) and (b), there are no coarse grains are found in Fig. 8(c) and 6(d), lots of recrystallized grain are generated under these deformation conditions. Initial casting structure has almost completely been replaced by forging structure through complete dynamic recrystallization. Fig. 8(e) and (f) shows that complete dynamic recrystallization has taken place under these deformation conditions, and the recrystallized grains grow obviously. Although that is not obvious, it still can be observed that the grain size of Fig. 8(f) is slightly larger than that of Fig. 8(e). This is due to the lower strain rate allowing the recrystallized grains to gain more time for growth.

Through comprehensive analysis of microstructures and deformation conditions, it can be found that as-cast 40CrNiMo occurs incomplete dynamic recrystallization at low deformation temperature and high strain rate. With the increase of deformation temperature or the decrease of strain rate, it makes the transition to complete dynamic recrystallization. The recrystallized grains significantly grow at high deformation temperature and low strain rate.

3.4. Kinetic model of DRX

Kinetic model of DRX proposed by Sellars is adopted in this study, including peak strain equation and critical strain equation [27,28], as follow.

\[
\begin{align*}
\epsilon_p &= a_1 2^{m_1} = a_1 \left( \epsilon \exp \left[ \frac{Q_1}{RT} \right] \right)^{m_1} \\
\epsilon_c &= k \epsilon_p
\end{align*}
\]  

(6)

Where \( \epsilon_p \) is the peak strain, \( \epsilon_c \) is the critical strain for DRX, \( \epsilon \) is the strain rate (s\(^{-1}\)), \( T \) is the deformation temperature (K), \( R \) is the gas constant, \( Q_1 \) is the recrystallization activation energy (J·mol\(^{-1}\)), \( a_1, m_1 \) and \( k \) are material constants.

Taking the logarithm of both sides of the expression of \( \epsilon_p \) in the Eq. (6) gives

\[
\ln \epsilon_p = \ln a_1 + m_1 \ln \epsilon + m_1 Q_1/(RT)
\]  

(7)

The peak strains under deformation temperatures at 900°C–1100°C with strain rates of 0.001 s\(^{-1}\)–0.1 s\(^{-1}\) are listed in Table 2. Based on these data, the material constants are resolved using linear regression method. The linear fitting curves for each material constant are shown in Fig. 9(a)–(c). The values are: \( m_1 = 0.1577 \), \( a_1 = 0.005 \), \( Q_1 = 311.6 \) kJ/mol.

The expression of \( \epsilon_p \) is shown in Eq. (8). Fig. 9(d) shows the linear fitting curve of the measured versus the predicted \( \epsilon_p \).

<table>
<thead>
<tr>
<th>Strain rate (s(^{-1}))</th>
<th>Deformation temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>900, 1000, 1100</td>
</tr>
<tr>
<td>0.01</td>
<td>0.2728, 0.1614, 0.1297</td>
</tr>
<tr>
<td>0.1</td>
<td>0.3342, 0.2366, 0.1634</td>
</tr>
<tr>
<td></td>
<td>0.5393, 0.3665, 0.2554</td>
</tr>
</tbody>
</table>

Table 2 – Values of peak strain under different deformation conditions.
The R square is 0.982, which reveals that the expression of \( \varepsilon_p \) has a high accuracy.

\[
\varepsilon_p = 0.005 \sqrt{\exp \left( \frac{3.116 \times 10^5}{RT} \right)} \times 1.577
\]  

Critical strain as a criterion for dynamic recrystallization can be obtained by the relation curve of work hardening rate \( \dot{\theta} \) versus stress \( \sigma \). The work hardening rate \( \dot{\theta} \) reflects the change rule of the stress change rate with stress. According to the five stage work hardening theory, in the process of metal hot deformation, the relation curve of work hardening rate versus stress can be divided into (I) easy slip stage, (II) linear hardening stage, (III) dynamic recovery stage, (IV) large strain hardening stage, and (IV) dynamic recrystallization softening stage [29,30], as shown in Fig. 10.

There is an obvious inflection point in the work hardening curve from stage IV to stage V. The stress at this inflection point is the critical stress \( \sigma_c \). After the inflection point, hot deformation enters into the dynamic recrystallization stage. Work hardening rate \( \dot{\theta} \) reaches zero twice, the first time is when the stress reaches peak \( \sigma_p \), the second time is when stress reaches steady state \( \sigma_{ss} \). From \( \sigma_p \) to \( \sigma_{ss} \), the values of \( \dot{\theta} \) are negative, this is because the dynamic recrystallization softening plays a dominant role in this stage. Assume that if dynamic recrystallization does not occur, dynamic recovery is the only softening mechanism, \( \dot{\theta} \) will maintain a linear decline after reach the critical stress \( \sigma_c \) until it reaches zero. At this point, the dynamic recovery softening and work hardening strengthening reach a balance, stress reaches the saturated stress \( \sigma_s \).

In the work hardening curve, the inflection point from stage IV to stage V is the initial point of DRX, so the mathematic relation between \( \dot{\theta} \) and \( \sigma \) at this point can be express as follow.

\[
\frac{\partial^2 \theta}{\partial \sigma^2} = 0
\]  

Where, the formula of work hardening rate \( \dot{\theta} \) is

\[
\dot{\theta} = \frac{d \sigma}{d \varepsilon}
\]  

---

**Fig. 8 – Microstructures of as-cast 40CrNiMo after isothermal compression.**
According to Eq. (11), the critical strain of DRX can be obtained by the ln $\theta - \varepsilon$ curve. Based on the research of Mirzadeh and Najafizadeh, ln $\theta$ can be expressed by a cubic polynomial of $\varepsilon$ [31, 32].

$$\ln \theta = A_1 \varepsilon + A_2 \varepsilon^2 + A_3 \varepsilon^3 + A_4 \varepsilon^{1/2} + A_5 \varepsilon^{1/3} \quad (0 \leq \varepsilon \leq \varepsilon_p) \quad (12)$$

So, the critical strain of dynamic recrystallization can be expressed as

$$\varepsilon_c = -\frac{A_2}{3A_3} \quad (13)$$

The cubic polynomial fitting curves of ln $\theta - \varepsilon$ at deformation temperature of 1000°C are shown in Fig. 11. The critical strains under different deformation conditions are listed in Table 3. According to the data in Tables 2 and 3, the material constant k in Eq. (6) can be ascertained by the linear fitting curve of $\varepsilon_p - \varepsilon_c$, as shown in Fig. 12, the slope is 0.5348, and the R square is 0.908.

**Table 3 – Values of critical strain under different deformation conditions.**

<table>
<thead>
<tr>
<th>Strain rate $(s^{-1})$</th>
<th>Deformation temperature $(^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>1100</td>
</tr>
<tr>
<td>0.001</td>
<td>0.0726</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>0.0456</td>
</tr>
<tr>
<td>0.01</td>
<td>0.1649</td>
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<td></td>
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<td></td>
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<td>0.1</td>
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<td></td>
<td>0.1572</td>
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<td>0.0709</td>
</tr>
</tbody>
</table>

**Fig. 9 – Linear fitting curves for equation of $\varepsilon_p$.**

**Fig. 10 – Relation curve of work hardening rate versus stress.**
3. Kinematic model of DRX

Dynamic recrystallization volume fraction model is an important part of kinematic model. Through the metallographic analysis to determine the DRX volume fraction will be a difficult and tedious process. Therefore, the method presented by Sellars is adopted, the DRX volume fraction can be ascertained by the flow stress-strain curve [33–36]. The DRX volume fraction \( X_{\text{DRX}} \) can be expressed as follows:

\[
X_{\text{DRX}} = \frac{\sigma - \sigma_s}{\sigma_s - \sigma_{ss}} \sqrt[3]{\frac{1}{2}} \sqrt[3]{\frac{1}{2}} \left( \epsilon \geq \epsilon_c \right)
\]

(15)

where \( \sigma_s \) is the saturated stress, \( \sigma_{ss} \) is the steady stress, \( \sigma \) is the flow stress. \( \epsilon \) can be obtained from the true stress-strain curve, \( \sigma_s \) and \( \sigma_{ss} \) can be obtained from the work hardening rate-stress curve. And then \( X_{\text{DRX}} \) under different deformation conditions can be calculated.

The work hardening rate-stress curves at deformation temperature of 1100° are shown in Fig. 13. The saturated stress \( \sigma_s \) and steady stress \( \sigma_{ss} \) under different deformation conditions are listed in Table 4. Based on the data in the Table 4, the stress values corresponding to 50% dynamic recrystallization is calculated by using Eq. (15). Then the corresponding strain \( \epsilon_{0.5} \) can be determined by the true stress-strain curve. The values of \( \epsilon_{0.5} \) under different deformation conditions are listed in Table 5.

On the basis of Avrami recrystallization theory, the kinematic model of DRX can be described as follows:

\[
\begin{align*}
X_{\text{DRX}} &= 0.5 \sqrt[3]{\frac{1}{2}} \sqrt[3]{\frac{1}{2}} \frac{1}{2} \left( 0 \leq \epsilon \leq \epsilon_c \right) \\
X_{\text{DRX}} &= 1 - \exp \left[ -\beta_d \left( \frac{\epsilon - \epsilon_c}{\epsilon_{0.5}} \right)^{k_d} \right] \sqrt[3]{\frac{1}{2}} \sqrt[3]{\frac{1}{2}} \frac{1}{2} \left( \epsilon \geq \epsilon_c \right) \\
\epsilon_{0.5} &= a_2 \epsilon^{m_2} \exp \left[ Q_d / (RT) \right]
\end{align*}
\]

3.5. Kinematic model of DRX

So, the kinetic model of dynamic recrystallization of as-cast 40CrNiMo alloy steel can be expressed as

\[
\begin{align*}
\epsilon_p &= 0.005 \epsilon \exp \left[ 3.116 \times 10^5 / (RT) \right]0.1577 \\
\epsilon_c &= 0.5348 \epsilon_p
\end{align*}
\]

(14)

Fig. 11 – Cubic polynomial fitting curves of \( \ln \theta - \epsilon \).

Fig. 12 – Linear fitting curve of \( \epsilon_p - \epsilon_c \).

Fig. 13 – Work hardening rate-stress curves.

Table 4 – Values of saturated stress and steady stress under different deformation conditions (MPa).

<table>
<thead>
<tr>
<th>Strain rate (s(^{-1}))</th>
<th>Deformation temperature (°C)</th>
<th>( \sigma_s )</th>
<th>( \sigma_{ss} )</th>
<th>( \sigma_s )</th>
<th>( \sigma_{ss} )</th>
<th>( \sigma_s )</th>
<th>( \sigma_{ss} )</th>
</tr>
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<tbody>
<tr>
<td>900</td>
<td>53.2</td>
<td>62.65</td>
<td>34.1</td>
<td>42.99</td>
<td>22.03</td>
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<td></td>
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<tr>
<td>1000</td>
<td>75.6</td>
<td>94.5</td>
<td>49.76</td>
<td>64.32</td>
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<td>1100</td>
<td>122.2</td>
<td>127.4</td>
<td>76.8</td>
<td>95.08</td>
<td>45.93</td>
<td>58.48</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 – Values of \( \epsilon_{0.5} \) under different deformation conditions.

<table>
<thead>
<tr>
<th>Strain rate (s(^{-1}))</th>
<th>Deformation temperature (°C)</th>
<th>( \epsilon_{0.5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>0.3187</td>
<td>0.2206</td>
</tr>
<tr>
<td>1000</td>
<td>0.5074</td>
<td>0.3257</td>
</tr>
<tr>
<td>1100</td>
<td>0.6698</td>
<td>0.4070</td>
</tr>
<tr>
<td>0.001</td>
<td>0.3187</td>
<td>0.2206</td>
</tr>
<tr>
<td>0.01</td>
<td>0.5074</td>
<td>0.3257</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6698</td>
<td>0.4070</td>
</tr>
</tbody>
</table>
Where $X_{\text{DRX}}$ is the DRX volume fraction, $\varepsilon_c$ is the critical strain, $\varepsilon_{0.5}$ is the strain for 50% dynamic recrystallization, $\dot{\varepsilon}$ is the strain rate (s$^{-1}$), $T$ is the deformation temperature (°), $R$ is the gas constant, $\beta_2$, $k_2$, $a_2$ and $m_2$ are the material constants, $Q_2$ is the activation energy for 50% dynamic recrystallization (kJ/mol).

Taking the logarithm of both sides of the expression of $\varepsilon_{0.5}$ in the Eq. (16) gives

$$\ln \varepsilon_{0.5} = \ln a_2 + m_2 \ln \dot{\varepsilon} + Q_2/(RT)$$  \hspace{1cm} (17)

Based on the data in the Table 5, the material constants are resolved through the linear regression analysis. The linear fitting curves for each material constant are shown in Fig. 14. The values are: $m_2 = 0.1534$, $a_2 = 0.0054$, $Q_2 = 50.514$ kJ/mol. The expression of $\varepsilon_{0.5}$ is shown in Eq. (18). Fig. 15 shows the linear fitting curve of the measured versus the predicted $\varepsilon_{0.5}$. The R square is 0.979, which reveals that the expression of $\varepsilon_{0.5}$ has a high accuracy.

$$\varepsilon_{0.5} = 0.0054\dot{\varepsilon}^{0.1534} \exp [5.0514 \times 10^4/(RT)]$$  \hspace{1cm} (18)

Taking the logarithm of both sides of the expression of $X_{\text{DRX}}$ in the Eq. (16) gives

$$\ln [-\ln (1 - X_{\text{XRD}})] = \ln \beta_2 + k_2 \ln \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}} \right)$$  \hspace{1cm} (19)

The relation between $X_{\text{DRX}}$ and $\sigma$ is ascertained by Eq. (15), then the relation between $X_{\text{DRX}}$ and $\varepsilon$ can be ascertained. Based on the data of $X_{\text{DRX}}$ and corresponding $\varepsilon$, the material constants in Eq. (19) are resolved through the linear regression analysis. Fig. 16 shows the linear fitting curves under the deformation temperature of 1000°C. The average slope of the curves under the different deformation conditions is 1.88097, and the average intercept is 0.23995. So the values are: $k_2 = 1.88097$, $\ln \beta_2 = 0.23995$ ($\beta_2 = 1.27119$). The expression of $X_{\text{DRX}}$ is shown in Eq. (20).

$$X_{\text{DRX}} = 1 - \exp \left[ -1.27119 \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}} \right)^{1.88097} \right] \quad (\varepsilon \geq \varepsilon_c)$$  \hspace{1cm} (20)

By combining Eqs. (18) and (20), the kinematic model of dynamic recrystallization of as-cast 40CrNiMo alloy steel can
be expressed as

\[
\begin{aligned}
X_{\text{DRX}} &= 0.5 \frac{1}{3} \frac{1}{3} \frac{1}{3} \left( 0 \leq \varepsilon \leq \varepsilon_0 \right) \\
X_{\text{DRX}} &= 1 - \exp \left[ -1.271187 \left( \frac{\varepsilon - \varepsilon_0}{\varepsilon_0.5} \right)^{1.88007} \right] \frac{1}{3} \frac{1}{3} \frac{1}{3} \left( \varepsilon \geq \varepsilon_0 \right) \\
r_{0,5} &= 0.0054^{0.1534} \exp [50513.73/(RT)] 
\end{aligned}
\]

(21)

In order to verify the accuracy of kinematic model, the values of \(X_{\text{DRX}}\) are calculated by using Eqs. (15) and (21) respectively under random deformation conditions. The correlation curve is shown in Fig. 17. The R square is 0.932, which reveals that the kinematic model has a high accuracy.

4. Conclusions

(1) The true stress-strain curves of as-cast 40CrNiMo alloy steel shows that the true stress increases with the increase of the strain rate, and it decreases with the increase of deformation temperature. The activation energy of experimental as-casting 40CrNiMo alloy steel is 332.1kJ/mol, that is less than as-rolled 40CrNiMo. The Arhenius type constitutive equation with Zener–Hollomon parameter for as-cast 40CrNiMo alloy steel can be expressed as follow.

\[
\begin{aligned}
\sigma &= \frac{1}{9.733 \times 10^{-3}} \ln \left( \frac{Z}{5.544 \times 10^{12}} \right)^{1/5.147} \\
&+ \left[ \frac{Z}{5.544 \times 10^{12}} \right]^{2/5.147} + 1 \right]^{1/2} \\
Z &= \dot{\varepsilon} \exp \left( \frac{3.321 \times 10^{5}}{8.3147} \right) 
\end{aligned}
\]

(2) With the increases of deformation temperature or the decreases of strain rate, the true stress-strain curves of as-cast 40CrNiMo gradually transform from dynamic recovery type to dynamic recrystallization type. The true stress-strain curves present the dynamic recrystallization type under the deformation temperatures at 900°C–1100°C, with strain rates of 0.001 s⁻¹–0.1 s⁻¹. The recrystallization activation energy is 311.6 kJ/mol. The DRX kinetic model of as-cast 40CrNiMo alloy steel can be expressed as follow.

\[
\begin{aligned}
\dot{\varepsilon} &= 0.005[3.116 \times 10^5/(RT)] \exp[\frac{271187}{RT}] \\
\varepsilon_{0.5} &= 0.534\varepsilon_p 
\end{aligned}
\]

(3) Within the range of deformation conditions where dynamic recrystallization occurs, as-cast 40CrNiMo occurs incomplete dynamic recrystallization at low deformation temperature and high strain rate. With the increase of deformation temperature or the decrease of strain rate, it makes the transition to complete dynamic recrystallization. The recrystallized grains grow significantly at high deformation temperature and low strain rate. The DRX kinetic model of as-cast 40CrNiMo alloy steel can be expressed as follow.

\[
\begin{aligned}
X_{\text{DRX}} &= 0 \quad (0 \leq \varepsilon \leq \varepsilon_c) \\
X_{\text{DRX}} &= 1 - \exp \left[ -1.271187 \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_0.5} \right)^{1.88007} \right] \frac{1}{3} \frac{1}{3} \frac{1}{3} \left( \varepsilon \geq \varepsilon_c \right) \\
r_{0.5} &= 0.0054^{0.1534} \exp[50513.73/(RT)] 
\end{aligned}
\]

Conflicts of interest

The authors declare no conflicts of interest.

Acknowledgements

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