Original Article

Transient dynamic analysis of laminated shallow spherical shell under low-velocity impact

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ABSTRACT

The impact of transient response laminated shallow shells due to transverse foreign object was investigated analytically and experimentally. The analytical analysis is presented by using the new higher order shear deformation shell theory. The contact force is assumed to be a known input to the analysis. Contact force and transverse deflection of laminated shell have been measured with a piezoelectric force transducer (load cell) and a piezoelectric bending transducer respectively. Laminates made from E-glass polyester and E-glass/carbon/polyester having different thickness and stacking sequences have been used. The analysis can be used to calculate the difference between the transient stresses, displacements, velocities, accelerations and energy observation at the center of laminated shell are plotted as function of time during impact between E-glass/polyester and E-glass/carbon/polyester by MATLAB code at different impact heights to determine whether they have a good resistant ability to impact. Finally, it was compared with the results of an experimental test to verify the accuracy and reliability of the theoretical analysis.

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1. Introduction

Composites are often used in circumstances concerning with the sudden application of loads such as sports goods, industrial, aerospace, automobiles, and civil engineering applications. The dynamic response of the structure ensues after load application and a state of stress may be generated that leads to failure. It is necessary to understand the response characteristics of the material with respect to its vital parameters including geometry, boundary conditions, and loading. Dobyns [1] studied the transient analysis for thin, simply supported composite plate under uniformly distributed load. Four types of loads (sine, triangular, rectangular and blast) were subjected over a rectangular surface area. The transient analysis was developed by the Laplace transform method. The contact force, displacement and strain results were compared with the work of other researches. Ramkumar and Thakar [2] studied the transient response of a thin, curved laminated plate comprising simple boundary condition subjected to impact loading. The contact force between the plate and the impactor is assumed to be a known input to the analysis. Fourier series expansion has been used to solve the governing equations, these equations comprise Airy stresses function and shell deformation. Impact test results for flat plates and closed-form analytical solutions for plate deflections are compared. Qian and Swanson [3] compared the Laplace transform analytical method with other technique that is based on a Rayleigh-Ritz approach with numerical integration in time in order to solve impact load for laminated plate. Cristoforou and Swanson [4] obtained an analytical
solution to the impact problem for the simply supported laminated plate, using the Fourier series expansion compounded with the Laplace transform technique. However, this method requires the contact deformation law to be linearized. Lin and Lee [5] investigated the low-velocity impact response of laminated plates by using the layer-wise theory. The layer-wise theory has been compared with different plate theories viz; classical plate, first order and third order shear deformation. The model superposition technique was employed to develop a forced vibration analysis. The different models have been introduced for the representation of the impact load, in which the contact area was assumed to be known. Two different interpolation techniques namely Lagrangian and Hermite were used for solving the nonlinear integral equation that resulted in the time history of the impact load. Christoforou and Swanson [6] employed finite element method by comparing the higher-order shear deformation theories (e.g., HST9, HST11, HST12) and the first-order shear deformation theory (FST) in order to investigate the impact behavior of doubly-curved composite sandwich shells. The finite element method consists of the nine nodded quadrilateral elements of the Lagrange techniques. The effect of core on the impact shell behavior was examined, change in response was detected as the thickness of core increases. Laura [7] analyzed the laminated open cylindrical shells subjected to impact loading. A spring-mass model was developed to determine the contact force between the shell and impactor during impact. The contact force was found to be dependent on material properties, mass of the shell, impactor, and the impact velocity. An analytic solution comprised of both contact deformation and transverse shear deformation was obtained. This analysis considered the effect of different parameters such as varying impact conditions, shell size and curvature on the contact force and central deflection of the shell. Nosier et al. [8] investigated numerical and theoretical analysis for the response of a simply-supported composite plate subjected to an impact load and both the analysis utilized Fourier series expansion for the solution of the dynamic plate equations. The Newmark integration method was employed to solve the nonlinear integral equation. The analytical formulation adopted the Laplace transform technique, requiring a linearization of the contact deformation and consequently to determine a linear contact stiffness. The contact force, displacement and strain results were compared for a squared and a rectangular plate. Nosier [9] investigated analytical and experimental transverse impact load on plain weave glass/epoxy composite plates with different thicknesses. The results showed that the absorbed impact energy had a non-linear relationship. When the thickness of plates was increased, the absorbed impact energy by the deflection decreased whereas the local indentation increased. Carvalho and Guedes Soares [11] examined transverse impact load of plain weave E-glass–epoxy laminates with various impactor mass and incident impact velocity but with the same incident impact energy of 24.43J. The unsupported dimensions of the specimens during the impact loading was (127*127) mm with clamped boundary condition. The thickness of the plate was 5 mm. Post-impact compression testing of impacted specimens was carried out using compression after impact test fixtures as per NASA 1142 standard. From impact test results, the damage tolerance was higher for low mass and high-velocity combination as compared to high mass and low-velocity combination. Lee and Lee [13] investigated the impact and compression after impact characteristics of a typical plane weave fabric E-glass/epoxy composite for plates with different thicknesses and the same incident impact energy. Post-impact-compression testing of impacted specimens has been carried out using compression after impact test fixture as per NASA 1142 standard. The peak contact force, maximum plate displacement and the residual compressive strength was presented as a function of plate thickness. Also, the damage mechanism was studied during the impact test. Naik et al. [18] investigated from the dynamic simulation for fiber composite laminates and analysis was carried out based on ANSYS ACP module and Explicit Dynamics module. The damage on the material was observed according to the change in the maximum stress and the strain at the top surface of the samples. The difference between the stress and strain of the carbon fiber and the glass fiber at different impact heights was considered to determine the resistant ability to impact. The numerical results were compared with the results of drop weight test. Soedel [19] investigated the behavior of laminated composite cylinder under impact loading by using Hertzian contact theory and numerical simulation (Abaqus software analyzer). Different design parameter effects were considered such as number of layers, thickness of laminated mass of impactor and height of impact. The impactor hit the cylinder at different points in the lateral and longitudinal direction with velocity (2 m/s). The results of experimental work were compared with those obtained from the numerical solution. In the transient response problem, the temporal variation of the contact force between the shallow spherical shell and the impactor was obtained from impact test data. The contact force was assumed to act at the center point of load and its boundaries were assumed to be simple supports. The governing equations for simply supported orthotropic shallow shells were solved using new higher order shear deformation [20].

2. Solution procedure

The new displacement field used in the present study is [11]:

\[
\begin{align*}
\hat{u}(x_1,x_2,z,t) &= \left(1 + \frac{z}{R_1}\right)u((x_1,x_2,t) - z \frac{\partial w}{\partial x_1} + z\sqrt{2\eta^2(\xi^2)} \omega_1 \\
\hat{v}(x_1,x_2,z,t) &= \left(1 + \frac{z}{R_2}\right)v((x_1,x_2,t) - z \frac{\partial w}{\partial x_2} + z\sqrt{2\eta^2(\xi^2)} \omega_2 \\
\hat{w}(x_1,x_2,z,t) &= w((x_1,x_2,t)
\end{align*}
\]

Where (\eta = 2.85) and (\omega_1, \omega_2) are the displacements along the orthogonal curvilinear coordinates such that the z1 and z2 curves are lines of principal curvature on the mid surface z = 0, and z curves are straight lines perpendicular to the surface z = 0. The parameters R1 and R2 denote the values of the principal radii of curvature of the middle surface. All displace-
ment components \((u, v, w, \varphi_2\) and \(\varphi_3\) are functions of \((x_1, x_2)\) and time \(t\) as shown in Fig. 1.

The strain-displacement relations take the form [5]

\[
\varepsilon_1 = \frac{1}{A_1} \left( \frac{\partial u}{\partial x_1} + \frac{w}{R_1} \right), \quad \varepsilon_2 = \frac{1}{A_1} \left( \frac{\partial v}{\partial x_2} + \frac{w}{R_2} \right), \quad \varepsilon_3 = \frac{1}{A_2} \left( \frac{\partial w}{\partial x_2} + 1 \right) \\
\varepsilon_4 = \frac{A_2}{A_1} \left( \frac{\partial v}{\partial x_1} + A_1 \frac{\partial}{\partial x} \left( \frac{v}{A_2} \right) \right), \quad \varepsilon_5 = \frac{A_1}{A_2} \left( \frac{\partial w}{\partial x_2} + A_2 \frac{\partial}{\partial x} \left( \frac{w}{A_1} \right) \right) \\
\varepsilon_6 = \frac{A_2}{A_1} \frac{\partial v}{\partial x_1} + \frac{A_1}{A_2} \frac{\partial w}{\partial x_2} + \frac{\partial u}{\partial x_1} + \frac{\partial w}{\partial x_2}
\]

Where
\[
A_1 = \left( 1 + \frac{x_1}{R_1} \right), \quad A_2 = \left( 1 + \frac{x_2}{R_2} \right)
\]

Substituting Eq. (1) into Eq. (2), we obtained:

\[
\varepsilon_1 = \varepsilon_1^0 + \frac{d}{dx} \left[ \int \left( (1 - 4 * \log (m)) \cdot \left( \frac{z}{h} \right)^2 \right) \right] dx_1 \\\n\varepsilon_2 = \varepsilon_2^0 + \frac{d}{dx} \left[ \int \left( (1 - 4 * \log (m)) \cdot \left( \frac{z}{h} \right)^2 \right) \right] dx_2 \\
\varepsilon_3 = \varepsilon_3^0 + \frac{d}{dx} \left[ \int \left( (1 - 4 * \log (m)) \cdot \left( \frac{z}{h} \right)^2 \right) \right] dx_3
\]

According to Hamilton's Principles: The equation of motion of the new higher order theory will be derived using the dynamic version of the principle of virtual displacements [6]. The detailed theoretical analysis and mathematical principle can be seen in Refs. [18] & [24]:

\[
\int_{t_0}^{t_1} (\delta U + \delta V - \delta K) \, dt = 0
\]

Where

\[
\delta U = \int_A \int_2 \left( \varepsilon_{11} \cdot \delta \varepsilon_1 + \varepsilon_{22} \cdot \delta \varepsilon_2 + \varepsilon_{44} \cdot \delta \varepsilon_4 + \sigma_{55} \cdot \delta \varepsilon_5 \\
+ \sigma_{66} \cdot \delta \varepsilon_6 \right) \cdot A_1 \cdot A_2 \cdot d\varepsilon_1 \cdot d\varepsilon_2 \\
\]

Where:
\(\{N_i, M_i, P_i\}\) are the result of the following integration:

\[
\{N_i, M_i, P_i\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \varepsilon_{11} \left( \int \left[ (1 - 4 * \log (m)) \cdot \left( \frac{z}{h} \right)^2 \right] \right) \cdot A_1 \cdot A_2 \cdot d\varepsilon_1 \cdot d\varepsilon_2 (i = 1.2.6)
\]
\[ \{ Q_i, K_j \} = \int_{-\frac{1}{2}}^{1} \left( 1 - 4^* \log (m) * \left( \frac{Z}{h} \right)^2 \right) * m^{-2} f(y) \]  
\[ 'A1' * A2' \text{dz} \quad (i = 4.5) \quad (j = 1.2) \]

\[ \delta K = - \int_{-\frac{1}{2}}^{1} \int_{-\frac{1}{2}}^{1} \rho \left( \ddot{u} \dot{u} \dot{u} + \ddot{v} \dot{v} + \ddot{w} \dot{w} \right) * A1A2 \text{dzAdt} \quad (10) \]

The virtual work done by applied forces is:

\[ \delta V = - \int_{-\frac{1}{2}}^{1} \int_{-\frac{1}{2}}^{1} q^* \dot{w} A1A2 \text{dzAdt} \quad (11) \]

Now, substituting Eq. (11), Eq. (10) & Eq. (6) in to Eq. (5), we obtained

\[ \int_{-\frac{1}{2}}^{1} \int_{-\frac{1}{2}}^{1} \left( N_{12} \dddot{w} + M_{12} \dddot{v} + P_{12} \dddot{w} + N_{2} \dddot{u} + M_{2} \dddot{u} + P_{2} \dddot{u} \right. \]
\[ + N_{6} \dddot{w} + M_{6} \dddot{v} + K_{6} \dddot{v} + P_{6} \dddot{w} + K_{6} \dddot{v} - q \dddot{w} \]
\[ + (l_{3} \dddot{u} + l_{3} \dddot{v} + l_{2} \dddot{w}) \dddot{u} + (l_{3} \dddot{u} + l_{3} \dddot{v} + l_{2} \dddot{w} - l_{2} \dddot{w}) \dddot{v} \]
\[ + (l_{3} \dddot{u} + l_{3} \dddot{v} + l_{2} \dddot{w}) \dddot{u} + (l_{3} \dddot{u} + l_{3} \dddot{v} + l_{2} \dddot{w} - l_{2} \dddot{w}) \dddot{v} \]
\[ \delta w = (l_{3} \dddot{u} + l_{3} \dddot{v} + l_{2} \dddot{w}) \dddot{u} + (l_{3} \dddot{u} + l_{3} \dddot{v} + l_{2} \dddot{w} - l_{2} \dddot{w}) \dddot{v} \]
\[ \text{dx1dx2} dt = 0 \quad (12) \]

The result forces are given by:

\[ \begin{align*}
N_1 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{11} \right) dz \\
M_1 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{22} \right) dz \\
N_2 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{66} \right) dz \\
M_2 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{11} \right) dz \\
N_6 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{22} \right) dz \\
M_6 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{66} \right) dz
\end{align*} \]

and

\[ \begin{align*}
K_1 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{44} \right) f(z) dz \\
P_1 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{22} \right) f(z) dz \\
K_2 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{66} \right) f(z) dz \\
P_6 &= \sum_{k=1}^{n} \int_{-\frac{1}{2}}^{1} \left( \sigma_{11} \right) f(z) dz
\end{align*} \]

The plane stress reduced stiffness is:

\[ Q_{11} = \frac{E_1}{1 - \mu_{12} \mu_{21}} \quad Q_{12} = \frac{\mu_{12} E_1}{1 - \mu_{12} \mu_{21}} \quad Q_{22} = \frac{E_2}{1 - \mu_{12} \mu_{21}} \]

\[ Q_{66} = G_{12} \quad Q_{44} = G_{23} \quad Q_{55} = G_{31} \quad (15) \]

From the constitutive relation of the kth lamina the transformed stress-strain relation of an orthotropic lamina in a plane state of stress are:

\[ \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{66} \\ \sigma_{44} \\ \sigma_{55} \end{bmatrix} = \begin{bmatrix} 11W \\ Q_{11} \quad Q_{12} \quad Q_{16} \quad 0 \quad 0 \\ Q_{12} \quad Q_{22} \quad Q_{26} \quad 0 \quad 0 \\ Q_{16} \quad Q_{26} \quad Q_{66} \quad 0 \quad 0 \\ 0 \quad 0 \quad 0 \quad Q_{44} \quad 0 \\ 0 \quad 0 \quad 0 \quad 0 \quad Q_{55} \end{bmatrix} \begin{bmatrix} \epsilon_{1} \\ \epsilon_{2} \\ \epsilon_{6} \\ \epsilon_{4} \\ \epsilon_{5} \end{bmatrix} \]

By substituting the stress–strain relations into the definitions of force and moment resultants of the present theory given in Eq. (7) the following constitutive equations are obtained:

\[ \begin{bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \\ P_1 \\ P_6 \end{bmatrix} = \begin{bmatrix} A_{11} \quad A_{12} \quad A_{16} \quad B_{11} \quad B_{12} \quad B_{16} \quad E_{11} \quad E_{12} \quad E_{16} \\ A_{12} \quad A_{22} \quad A_{26} \quad B_{12} \quad B_{22} \quad B_{26} \quad E_{12} \quad E_{22} \quad E_{26} \\ A_{16} \quad A_{26} \quad A_{66} \quad B_{16} \quad B_{26} \quad B_{66} \quad E_{16} \quad E_{26} \quad E_{66} \\ B_{11} \quad B_{12} \quad B_{16} \quad E_{11} \quad E_{12} \quad E_{16} \\ B_{12} \quad B_{22} \quad B_{26} \quad E_{12} \quad E_{22} \quad E_{26} \\ B_{16} \quad B_{26} \quad B_{66} \quad E_{16} \quad E_{26} \quad E_{66} \\ E_{11} \quad E_{12} \quad E_{16} \quad F_{11} \quad F_{12} \quad F_{16} \\ E_{12} \quad E_{22} \quad E_{26} \quad F_{12} \quad F_{22} \quad F_{26} \quad H_{12} \quad H_{22} \quad H_{26} \\ E_{16} \quad E_{26} \quad E_{66} \quad F_{16} \quad F_{26} \quad F_{66} \quad H_{16} \quad H_{26} \quad H_{66} \quad \end{bmatrix} \begin{bmatrix} \epsilon_{1} \\ \epsilon_{2} \\ \epsilon_{6} \\ \epsilon_{4} \\ \epsilon_{5} \end{bmatrix} \]
\[
\begin{bmatrix}
K_1 \\
K_2
\end{bmatrix}
= \begin{bmatrix}
L_{44} & L_{45} \\
L_{45} & L_{55}
\end{bmatrix}
\begin{bmatrix} \varepsilon_3^2 \\
\varepsilon_4^2
\end{bmatrix}
\]
(18)

Where

\[A_{ij} = \int_{-h/2}^{h/2} Q_{ij} \, dz \quad i = 1, 2, 4, 5, 6\]

\[B_{ij} = \int_{-h/2}^{h/2} E_{ij} H_{ij} \, dz = \int_{-h/2}^{h/2} Q_{ij} \left[ z \, z^2 \, zm^{-2}(\varepsilon_3^2)^2 \cdot z^2 \cdot m^{-2}(\varepsilon_3^2)^2 \right] \, dz\]

\[i = 1, 2, 6\]

\[L_{ij} = \int_{-h/2}^{h/2} \left( \int_{z/2}^{z} \left( 1 - \log(m) \left( \frac{z}{R} \right)^2 \right) \right) \, dz\]

\[I_1, I_2, I_3, I_4, I_5, I_6 = \int_{-h/2}^{h/2} \frac{1}{2} \left( 1 - \log(m) \left( \frac{z}{R} \right)^2 \right) \, dz\]

\[\text{(20)}\]

The Navier solution exists if the following stiffness's are zero for symmetric and anti-symmetric simply supported cross ply: Ai6 = Bi6 = Di6 = Ei6 = Hi6 = A45 = L45 = F16 = 0 (i = 2, 6). The simply supported boundary conditions are assumed to be of the form:

\[\text{At } x_1 = 0, a : v = w = N1 = M1 = P1 = \varphi_2 = 0\]

and at

\[x_1 = 0, b : u = w = N2 = M2 = P2 = 1 \varphi_1 = 0\]

(21)

(22)

Now, we used the separation of variables technique in order to calculated transient behavior for laminated spherical shallow shell. This method assume solution to equations of motion in the form:

\[u(x_1, x_2) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos(\alpha x_1) \sin(\beta x_2) T_{mn}(t)\]

\[v(x_1, x_2) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin(\alpha x_1) \cos(\beta x_2) T_{mn}(t)\]

\[w(x_1, x_2) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin(\alpha x_1) \sin(\beta x_2) T_{mn}(t)\]

\[\varphi_1(x_1, x_2) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn} \cos(\alpha x_1) \sin(\beta x_2) T_{mn}(t)\]

(23)

(24)

Where \(\alpha = n^{+} \frac{a}{\beta} + m^{+} \frac{b}{\beta}\) Ann, Bmn, Cmn, Dmn, Emin are arbitrary constants. The orthogonality condition of principle modes can be established with the result as shown below:

\[\langle \omega_{mn}^2 - \omega_{jr}^2 \rangle \int_{0}^{a} \int_{0}^{b} \left[ (I_1 A_{mn} - \alpha I_2 C_{mn} + I_4 D_{mn}) A_{rs} + (I_1 B_{mn} - \beta I_2 C_{mn} + I_4 E_{mn}) B_{rs} + (I_1 A_{mn} + \beta I_2 C_{mn} - I_3 (\alpha^2 C_{mn} + \beta^2 C_{mn}) + I_5 (\alpha D_{mn} + \beta E_{mn}) + I_1 C_{mn}) C_{rs} + (I_4 A_{mn} - \alpha I_5 C_{mn} + I_4 D_{mn}) D_{rs} + (I_1 B_{mn} - \beta I_5 C_{mn} + I_4 E_{mn}) E_{rs} \right] dx_1 dx_2 = 0\]

The general distributed loads are expanded in a series of principal modes as shown

\[g_1 = \sum_{m=1}^{\infty} f_{mn}(t) (I_1 A_{mn} - \alpha I_2 C_{mn} + I_4 D_{mn})\]

\[g_2 = \sum_{m=1}^{\infty} f_{mn}(t) (I_1 B_{mn} - \beta I_2 C_{mn} + I_4 E_{mn})\]

\[q = \sum_{m=1}^{\infty} f_{mn}(t) \left( (I_1 A_{mn} + \beta I_2 B_{mn} - I_3 (\alpha^2 C_{mn} + \beta^2 C_{mn}) + I_5 (\alpha D_{mn} + \beta E_{mn}) + I_1 C_{mn}) \right)\]

\[m_1 = \sum_{m=1}^{\infty} f_{mn}(t) (I_4 A_{mn} - \alpha I_5 C_{mn} + I_4 D_{mn})\]

\[m_2 = \sum_{m=1}^{\infty} f_{mn}(t) (I_1 B_{mn} - \beta I_5 C_{mn} + I_4 E_{mn})\]

(25)

The generalized forces \(f_{mn}(t)\) are determined by taking use of orthogonality condition. Multiplying Eq. (25a) by Ann, (25b) by Bmn, Eq. (25c) by Cmn, Eq. (25d) by Dmn, Eq. (25e) by Emin and adding the results, integrating over the plane area, and taking into account Eq. (24) leads to the result:

\[f_{mn}(t) = \int_{0}^{a} \int_{0}^{b} \left( g_1 A_{mn} + g_2 B_{mn} + q C_{mn} + m_1 D_{mn} + m_2 E_{mn} \right) dx_1 dx_2 \]

\[N_{mn}\]

(26)

Where substituting Eq. (16) into equations of motion, taking into account Eq. (25), gives:

\[T_{mn} + \omega_{mn}^2 T_{mn} = f_{mn}\]

(27)
For any \((m, n)\). The solution to above Eq. (20) is given by:

\[
T_{mn} = \frac{1}{\omega_{mn}} \int_{0}^{t} f_{mn}(t) \sin \omega_{mn}(t - r) \, dr
\]  

(28)

The force-time traces obtained through impact test, when a 1.86 mm laminate is impacted by an impactor with a tip radius of 40 mm are shown in Fig. 2. The contact force is assumed to act center point load on center of Laminated shell, \(q(x', y', t) = 0 \delta (x_1 - x_1') + \delta (x_2 - x_2') \) \(F(t)\), \((m = n = 1)\), the formal solution to the unknown functions may be expressed as:

\[
\begin{pmatrix}
\frac{u}{u}
\frac{w}{\vartheta_1}
\frac{\vartheta_2}{u}
\end{pmatrix} = \sum_{m=n+1}^{\infty} \left( \frac{4\pi q_0}{\pi a a} \right) \sin \left( \frac{mn\pi x}{a} \right) \sin \left( \frac{mn\pi y}{b} \right)
\]

\[
\begin{pmatrix}
\frac{\vartheta_1}{u}
\frac{\vartheta_2}{w}
\end{pmatrix} = \sum_{k=1}^{5} \left( \frac{A_{mm}(k) / N_{mm}(k) \cos (\alpha x_1) \sin (\beta x_2)}{B_{mm}(k) \cos (\alpha x_1) \sin (\beta x_2)} \right) \sin \frac{mn\pi x}{a} \sin \frac{mn\pi y}{b}
\]

\[
\int_{0}^{t} F(t) \sin \omega_{mn}(k) (t - r) \, dr
\]  

(29)

Note that the solution in Eq. (29) is normalized with respect to \(C_{mn}(k)\), the coefficients in expansion of \(w\). Where:

\[
C_{mn}(k) = \int_{0}^{b} \int_{0}^{a} \left( \frac{E_{mm} A_{mm} + I_1 D_{mm} A_{mm}}{E_{mm} B_{mm} + I_1 E_{mm} B_{mm}} + \frac{1}{E_{mm} B_{mm}} \right) \sin \frac{mn\pi x}{a} \sin \frac{mn\pi y}{b}
\]

\[
\begin{align*}
&\left( -\alpha_2 C_{mm} A_{mm} - \beta_2 C_{mm} B_{mm} + (1\alpha^2 + I_1 \beta^2) C_{mm} - \alpha_3 D_{mm} A_{mm} \\
&- \beta I_3 E_{mm} C_{mm} + I_3 C_{mm} + (I_3 E_{mm} A_{mm} + I_3 E_{mm} A_{mm}) \sin \frac{mn\pi x}{a} \sin \frac{mn\pi y}{b}
\end{align*}
\]

(30)

With sine loading in time domain are used.

\[
1 - F(t) = \begin{cases} \\
\sin \frac{\pi t}{td} & 0 \leq t \leq td \\
0 & t > td
\end{cases}
\]

3. Experimental work

Laminated are manufactured from two types of materials E-glass and E-glass/carbon woven fabric with polyester resin a cross ply (2-layerd, 4-layerd and 6-layerd) as shown Table 1. All samples dimensions are (20’20) cm with different thickness chosen. This type of samples are laminated shallow spherical shells. These spherical shells must be constructed carefully. In order to building template we should be use double sheets of wood type Medium-density fiber board (MDF), it is usually of 230-610 mm thickness therefore mold should be paste the two sheets tightly to form the smooth surface of the doubly curved shell with radius of curvatures R = 50 cm by using three dimensions CNC machine. The template then coated by a layer with (20 cm × 20 cm) of woven fabrics dry rectangular sheets as shown in Fig. (3 A) so the remainder of the woven fabrics rolled into the back of the mold and governs at the back with adhesive tape. This process is repeated for each layer until the staking of woven fibers sheets complete

1) Gypsum embossing template, 2) steel handel, 3) Gypsum template coated with woven carbon fabric, 4) Vacuum Bag of polyvinyl alcohol (PVA), 5) Rein inside of PVA, 6) Metallic vacuum pipe, 7) Vacuum machine, 8) Input resin place.

The mechanical properties of the laminates are given in Table 2. All of these properties are measured in the mechanical laboratory. The mechanical testing is carried out by accordance to Test ASTM Method (D 3039 M). Tensile Testing Machine. Low velocity impact tests are conducted using a vertical drop weight testing machine developed in the department laboratory as shown Fig. (5 A). To measure the impact force history, a load cell type piezoelectric force transducer is used. The impact point is on the center of the curved plate. Impact tests are performed at increasing impact heights 2 cm to 3 cm and The impactor was manufactured of steel material with length of (5) cm and diameter (3.5) cm. The weight of impactor was (100.4) gm and the mass density was \(\frac{7967}{kg/m^3}\). The Young modulus of the impactor was (200) Gpa and Poisson ratio (0.3). The density of glass/polyester (volume fraction) \(\text{??g} = 0.6\) and E-glass/carbon/polyester (volume fraction of glass and carbon was) \(\text{??g} = 0.3\) was found to be \((1514 \text{kg/m}^3)\) and \((1415 \text{kg/m}^3)\) respectively.

The apparatus for this test is designed and manufactured to deal with specific impact tests, consist of the base of rectangular reing. Contains a binding location of the shallow shell (20’20) cm and in the upper part there is small rectangular frame connect the impactor rod (30 cm wide50 cm length) to reach to the point of upper surface for composite shallow shell by the impactor with load cell that connected with it and there is another rectangular frame between the upper and lower rectangular frames (30 cm wide50 cm length) a Place of measuring instrument. For these cases used in this experimental works the boundary conditions was S.S. on the frame to support the laminated spherical, Generally, Settings can be summarized as follows:

<table>
<thead>
<tr>
<th>Type of materials</th>
<th>No. of layers</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass/polyester</td>
<td>2</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.8</td>
</tr>
<tr>
<td>Carbon/E-glass/polyester</td>
<td>2</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.8</td>
</tr>
</tbody>
</table>
Fig. 2 – Variation of impact load with time between experimental, Ansys workbench and curve fitting, for cross ply anti-symmetric simply supported (0/90) E-glass/polyester.

Fig. 3 – (A) building of template: Male (1) and female (2) of 20 cm x 20 cm. (B) Free gypsum template, (C) coating with woven carbon fabric.

Table 2 – Mechanical properties of laminated shallow shell used in experimental test.

<table>
<thead>
<tr>
<th>Types of materials</th>
<th>Num.of layers</th>
<th>Thick-ness (mm)</th>
<th>Elastic modulus? ?1 = E2 Gpa</th>
<th>Shear modulus G12 Gpa</th>
<th>poisons ratio? ?12</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass/polyester</td>
<td>(0/90)</td>
<td>1.8</td>
<td>29.12</td>
<td>1.89</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0/90)2</td>
<td>2.78</td>
<td>31.2</td>
<td>2.45</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(0/90)3</td>
<td>3.8</td>
<td>34.96</td>
<td>2.87</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0/90)</td>
<td>1.8</td>
<td>60.26</td>
<td>2.85</td>
<td>0.31</td>
</tr>
<tr>
<td>E-glass/carbon/polyester</td>
<td>(0/90)2</td>
<td>2.78</td>
<td>65.5</td>
<td>3.153</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>(0/90)3</td>
<td>3.8</td>
<td>70.1</td>
<td>3.61</td>
<td>0.25</td>
</tr>
</tbody>
</table>

1 Connect the load cell to the amplifier.
2 Make sure that the connection the electric devices such as Load cell amplifier, with (power supply).
3 Connect the load cell to the probe and this probe connected to the digital Oscilloscope in ch1 and choose 1x.
4 Connect the piezoelectric to the digital oscilloscope to ch2.
Putting the laminated spherical shell in its place in the frame and bounding it to the frame.

Impactor pulls to the top at launch position in order to generate potential energy in the rod of impactor from the gravity of the impactor system.

Releasing the impactor in which allows the striker rod to crush the specimen (laminated spherical) at the desired center point surface. The specimen was located in the frame, and manufactured specially to meet simply supported boundary condition of laminated spherical shell. The impactor was oriented to the desired point of sample. To satisfy the same principle of the free drop weight oriented to the desired point of specimen. As, the impact was lifted by hand to appropriate drop height, then, releasing the impactor rod to crash the surface of tested sample. The data logger connect was started from the force sensor was sent to it. A program (wave analysis) calculates and plots force-time, displacement-time curves as shown in the Fig. 4 below:

4. Results and discussion

4.1. Contact force plots

The contact force of experimental work were compared with that obtained from the Ansys workbench.19 (explicit dynamic)
and the variation between them was of the order of 10%. The contact force results for composite (E-glass/polyester) and hybrid (E-glass/carbon/polyester) with different layered and shell thickness under (0.02 & 0.03) m impact heights presented in Figs. 6–11). The contact force measured indicates the resistance offered by the shallow shell during the impact event. The peak contact force was higher for hybrid and lower for composite. The laminated shell with higher thickness offers more resistance during the impact event. The main reason is explained that the stiffness of hybrid increased with increased mass (described by its thickness) of laminated shallow shell more than composite materials.

4.2. Displacement, velocity and acceleration of shells

Figs. 6–11 show the shallow shell displacement, velocity and acceleration plots under two impact heights (0.02 and 0.03) m. The central displacement plots were derived from the piezoelectric bending transducer and it was compared with analytical solution and the maximum variation between them...
Fig. 7 – The Comparative analysis for impact load, central displacement, velocity and acceleration of (0/90)2 laminated shell under 0.02 m impact height between (a) glass/polyester. (b) 2glass/2carbon/polyester.
Fig. 8 – The Comparative analysis for impact load, central displacement, velocity and acceleration of (0/90)3 laminated shell under 0.02 m impact height between (a) glass/polyester. (b) Glass/carbon/polyester.
Fig. 9 – The comparative analysis for impact load, central displacement, velocity and acceleration of (0/90) laminated shell under 0.03 m impact height between (a) glass/polyester. (b) Glass/carbon/polyester.
Fig. 10 – The comparative analysis for impact load, central displacement, velocity and acceleration of (0/90)2 laminated shell under 0.03 m impact height between (a) glass/polyester. (b) 2glass/2carbon/polyester.

was of the order of 14.6%. To calculate the central displacement experimentally we use the following Eqs. [16] and [20]:

\[ W_{ex}(t) = d_{13} L_a E_3(t) \]

\[ E_3(t) = \frac{\text{Voltage}(t)}{h_a} \]

Where: \( W_{ex}(t) \): displacement measured during experimental work (mm). \( L_a \): is the length of the beam actuator.
The comparative analysis for impact load, central displacement, velocity and acceleration of (0/90)3 laminated shell under 0.03 m impact height between (a) E-glass/polyester. (b) E-glass/carbon/polyester.

Voltage ($V(t)$): is the applied voltage. $h_a$: is the thickness of the beam actuator (0.46 mm). $d_{13}$: is Induced strain coefficient ($-210E-12 m/V$). $E_3 (t)$: is the electric polarization field (V).

The velocity plots were derived from the displacement plots while the acceleration plots were derived from the velocity plots also, the results compared with theoretical analysis. The results indicated that the value deflection, velocity and acceleration of laminated shell for hybrid are smaller than those for composite materials, because the structure of hybrid materials is large stiffer with increased shell thickness. Also,
4.3. Shell stresses and absorbed energy

The dimensionless stresses in the center of laminated shell plots derived analytically by using Eq.16.

\[
\sigma_1 = \frac{\sigma_{11} \left( \frac{a}{b}, \frac{b}{2}, \frac{h}{2} \right)}{q_0}, \quad \sigma_2 = \frac{\sigma_{22} \left( \frac{a}{2}, \frac{b}{2}, \frac{h}{2} \right)}{q_0}, \quad \sigma_4 = \frac{\sigma_{44} \left( \frac{a}{2}, 0, \frac{h}{2} \right)}{q_0}
\]

Where: \( \sigma_1 = \sigma_2 \) because the laminated shallow shell is woven type \( (E_1 = E_2, a = b \text{ and } R_x = R_y) \).

SEG.1 = SEG.4 = \( \sigma_4 \)

Where SEG.1: Normal stress (in x direction) & SEG.4 Shear stresses (in x-z direction).

Figs. 12, 14, and 16 indicate the effect number of layered and shell thickness on the dimensionless normal stresses and shear stresses which it is subjected to the center impact load between composite and hybrid materials. The dimensionless normal stresses decrease with increasing shell thickness, this is due to the fact that starching-bending stiffness increases
with shell thickness; as a result, the stresses vary inversely with shell thickness. It is obvious that the maximum value of dimensionless shear stresses in hybrid are more than composite because of the modules of elasticity $E_1$, shear modules $G_{12}$ and passion's ratio for hybrid is larger than that for composite as shown in the Figs. 13, 15 and 17).

While Table 3 explained the energy observation was calculated using integration under the area of curve relation between the contact force and displacement results obtained from load cell and piezoelectric bending transducer data respectively by using trapezoidal method. The value of maximum shell observation energy was slightly less than the incident impact energy. This was possibly due to the loss of energy due to friction of the impactor with the guide rails during the impact event. Here the trend is reversed from the force results when increased number of layered and shell thickness of specimens have small central displacement values but it have higher energy observation.
Table 3 – The comparative analysis for energy observation (\(j\)) of anti-symmetric cross ply laminated shell under (0.02 and 0.03) m impact heights between (a) E-glass/polyester, (b) E-glass/carbon/polyester.

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>Thickness (mm)</th>
<th>Materials</th>
<th>Energy observation ((j))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Impact height</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.02 m</td>
</tr>
<tr>
<td>(0/90) 1</td>
<td>1.8</td>
<td>E-glass/polyester</td>
<td>0.0312</td>
</tr>
<tr>
<td>(0/90) 2</td>
<td>2.78</td>
<td>E-glass/polyester</td>
<td>0.0387</td>
</tr>
<tr>
<td>(0/90) 3</td>
<td>3.8</td>
<td>E-glass/polyester</td>
<td>0.0407</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E-glass/polyester</td>
<td>0.0463</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E-glass/polyester</td>
<td>0.0512</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E-glass/polyester</td>
<td>0.0558</td>
</tr>
</tbody>
</table>

5. Conclusions

Based on the validation and parametric study the following conclusions have been stated and discussed below:

1. The validation study of experimental work and the (NHOSD) under impact load which it is assumed center point load for shallow shell have a good agreement.
2. The results show the hybrid material better than composite under impact test because the improved mechanical properties effect increased the stiffness and reliability to resistance impact load of laminated shallow shell.
3. From the analysis of results, Stiffer structures produce higher impact forces, smaller center deflections, velocities, accelerations, and shorter contact duration times.
4. The arching-bending stiffness and the orthotropic ratio effect are in fact very large on the dimensionless normal stresses and shear stresses while increasing the shell thickness and its effect is found in hybrid more than composite.

But the value of energy observation between hybrid and composite materials was approaching whereas increases thickness of laminated because the structural stiffness of hybrid larger than composite.

References

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[23] M. Mohammed Idan, 2017 Dynamic analysis of cylindrical composite material shells under transverse impact loading, the college of engineering university of Baghdad in partial fulfillment of the requirements for the degree of doctor of philosophy in mechanical engineering applied mechanics.