Original Article

Dynamic crack propagation in nano-composite thin plates under multi-axial cyclic loading

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A R T I C L E   I N F O

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A B S T R A C T

The main cause of failure in structural mechanics is Fatigue, because of cyclic loading which effects on the crack growth, so in this research, the dynamic crack growth for thin plates subjected to multi-axial cyclic loading (non –proportional) have been studied. The material of plates are pure Copper and reinforced with two types of CNTs with 6 vol %, Single Wall Carbon Nano Tubes Carboxylic (SWCNTs-COOH) and Multi-Wall Carbon Nano Tubes Carboxylic MWCNTs-COOH. It was used an efficient way for producing a Nano-composite material, with perfect dispersions of CNTs, to enhance the mechanical properties of copper and getting a homogeneous mixture of Copper and CNTs. For fracture mechanics part will consider firstly, the theoretical model by using the fracture mechanics modeling to predict the enhancement, reliability and operating life of materials and finding a-N curve. Secondly, in the experimental work, a new apparatus was designed to apply the cyclic shear load (repeated and zero based) and constant tension, Also we will find the critical stress intensity factor Kc for mode I and II experimentally, and all the parameter of Paris Law (c & m Paris Law’s constants), (AK the change in stress intensity factor) and (da/dN crack growth per cycle), to insert it in ABAQUS program to simulate the model and compare the results to verify this work. The speed of the crack propagation also found experimentally and analytically at the secondary stage of the crack life, for pure copper experimentally crack speed 0.0.0940 mm/min and 0.1248 mm/min analytically, For MWCNTs + Cu 0.083 mm/min experimentally and 0.08809 mm/min analytically, for SWCNTs + Cu 0.1217 mm/min experimentally and 0.135 mm/min analytically, but also it can be seen the difference between the three material in the crack initiation, where the crack starts in the pure copper earlier than MWCNTs + Cu and at last SWCNTs + Cu's crack will start. So it’s seen that the life of copper was increased around 10% with MWCNTs-COOH and 18% with SWCNTs-COOH.

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1. INTRODUCTION

Nano-composite materials have been widely used in many engineering fields such as mechanical and space structures...
When the material used as an element in a structure contains a crack, it is necessary to study the distribution of the stress area around the crack, as well as the crack spread, and the inertia procedure should be considered when the applied loads or crack length depend on time or cycles. Also, the most common phenomenon in experiments shows the crack growth rate is constant during the extension date except in the last unstable stage [3]. The physical defects that arise at the time of production or use of materials are unavoidable and therefore must be taken into account [4]. Microscopic defects can cause structures that are assumed to be safe to fail, as they grow over time. In the past, when a component of some structures showed a fault, it was either repaired or retired from service. These precedents are today in many cases considered unnecessary, can’t be imposed, or may be too costly. In fact, safety margins should be allocated to smaller structures, due to increased demand for energy and preservation of materials. On the other hand, detecting a defect in the structure does not mean that it is no longer safe to use anymore. In general, there is no standard test rig for cyclic load experiments, hence, researchers were established their rigs to be in full compliance with their requirements, noting that the results of two various test rigs are hard to be compared even if the experiments are carried out within same topics; so, manufacturing of such rigs are based on two main factors, the first is the configuration of the specimen and the second is a functionality [5]. On the other hand, the outcomes that observed regarding crack growth behavior of the coupon scale differ from that of an actual full scale [6]. In most cases of preexistent central cracks in the proposed specimens (that subjected to uniaxial or multiaxial loads) the crack grow at both sides (in the same specimen) non-uniformly regarding crack growth per cycles (da/dN) and/or crack growth direction (θc) as a consequence of material microstructure [7], [8] and [9]. One of the most features of mixed mode conditions is the propagation in a non-uniform manner concerning crack growth behavior [8]. In this work, the mechanics part plays a central role, as they provide useful tools that allow for the analysis of materials that show cracks.

The main objectives of this work are:

- Produce a Nano composite materials (Cu+CNTs) with an efficient way, high quality and quantity.
- Improve copper resistance to fracture by applying multi-axial cyclic loading.
- Crack speed determination to prevent the sudden failure.

2. THEORETICAL

This part will consider the dynamic crack growth fracture mechanics for our multi-axial cyclic loading mechanism. The crack was created by using a wire-cut machine as a rectangular shape with dimensions (15 mm) and its location at the end of shear load grippers, (35 mm from the end) as shown in Fig. 1. So in this paper will consider Griffith energy criterion that concerned with the energetics of a system defined by an infinitely large plate of brittle material containing a single sharp through-crack loaded [10], see Fig. 2(a) and (b).

The strain energy stored per unit volume due to a uniformly applied stress σ with cycling shear stresses τ is given by [10]:

\[ U_0 = \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \]  \hspace{1cm} (1)

The energy released per unit thickness is given by:

\[ U = \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi a^2 \times \frac{h}{H} \]

\[ = \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi a^2 \text{ for Plane Stress (2)} \]

For critical point of fracture is at max value of energy release [10]:

\[ G = \frac{aU}{\partial a} = \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \frac{\pi a}{2} \]  \hspace{1cm} (3)

The excess energy due to unstable fracture can expressed as:

\[ U_a = \int_{a_0}^{a} (G-R)da = -R (a_i - a_o) \]

\[ + \int_{a_0}^{a} \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi a/2 da \]  \hspace{1cm} (4)

For R = \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi a_c/2 \]

Substituting Eq. (5) in Eq. (4) gives

\[ U_a = \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi a_c/2 (a_i - a_o) \]

\[ + \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi/4 \left( a_i^2 - a_o^2 \right) \]

\[ = \left( \frac{\sigma^2}{2E} + \frac{2(1+v)}{E} \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2 \right) \pi/4 (a_i - a_o)^2 \]  \hspace{1cm} (6)

For calculation the kinetic opening displacement of the crack in thin plate for multi axial loading it has suggested that to calculate the effective stress that relates a complex state of stresses to an equivalent uniaxial stress given by [11] :

\[ \sigma_{eff} = \frac{1}{\sqrt{2}} \sqrt{\sigma^2 + \left( \tau \sin \left( \frac{\omega t}{2} \right) \right)^2} \]  \hspace{1cm} (7)

The effective stress is a good approximation for non-proportional multi-axial stress with two kinds of stress, one of them is constant (as σ in this case study) and the other one is
cycling stress (shear stress \( t \)). Then the vertical displacement of the crack \( v \) can be written as [12]:

\[
v = \frac{2\sigma_{\text{eff}}}{E} \sqrt{a^2 - x^2}
\]

(8)

Since \( x \) is a function of \( a \) it may be written \( x = Ca \) for \( 0 < C < 1 \) then:

\[
v = \frac{2\sigma_{\text{eff}}}{E} \sqrt{a^2(1 - C^2)} = C_1 \frac{\sigma_{\text{eff}} a}{E}
\]

(9)

Where \( C_1 = 2\sqrt{1 - C^2} \)

So when the crack propagates then the displacement \( (v) \) will be various with time and become:

\[
\frac{\partial v}{\partial t} = \frac{C_1}{E} \frac{\partial (\sigma_{\text{eff}} a)}{\partial t}
\]

(10)

\[
= \frac{C_1}{E} \left( \frac{\partial \sigma_{\text{eff}}}{\partial t} a + \frac{\partial a}{\partial t} \sigma_{\text{eff}} \right)
\]

(11)

\[
\frac{\partial \sigma_{\text{eff}}}{\partial t} = \partial \left( \frac{1}{\sqrt{2}} \sqrt{\sigma^2 + (t \sin \frac{\omega t}{2})^2} \right) / \partial t
\]

(12)

From the dynamic condition for the crack growth, the kinetic energy of the displacement of the crack is:

\[
T = \frac{1}{2} \int \int \left( \frac{\partial v}{\partial t} \right)^2 \, dx \, dy
\]

(15)

\[
= \frac{1}{2} E \frac{1}{\rho} \int \int \left( \frac{\omega^2 \sin \omega t}{2 \sqrt{2} \sqrt{\sigma^2 + (t \sin \frac{\omega t}{2})^2}^2} \right) \, dx \, dy
\]

(16)
In a semi-infinite plate it is found experimentally that the integral of $C_1^2$ is equal to $ka^2$ thus Eq. (16) could be written as [12]:

$$T = ka^2 \frac{1}{2 E} \left[ \frac{\omega t^2 \sin \omega t}{2 \sqrt{2} \sqrt{\sigma^2 + (\sin(\frac{\omega t}{2}))^2}} \right]$$

$$+ \frac{\omega a \frac{1}{\sqrt{2}}}{\sqrt{2} \sqrt{\sigma^2 + (\sin(\frac{\omega t}{2}))^2}}$$

(17)

The critical value for the crack length to be unstable and begin to grow when the strain energy $U_e$ become equal or excess the kinetic energy $T$ then from Eq. (6) and (13):

$$\left( \frac{\sigma^2}{2E} + \frac{2(1+\mu)}{E} (\sigma^2 \sin^2 \alpha) \right) \pi/4(a_1 - a_2)ka^2 \frac{1}{2E} \left[ \frac{\omega t^2 \sin \omega t}{2 \sqrt{2} \sqrt{\sigma^2 + (\sin(\frac{\omega t}{2}))^2}} \right]$$

$$+ \frac{\omega a \frac{1}{\sqrt{2}}}{\sqrt{2} \sqrt{\sigma^2 + (\sin(\frac{\omega t}{2}))^2}}$$

(18)

$$ka^2 \frac{1}{2E} \left[ \frac{\omega t^2 \sin \omega t}{2 \sqrt{2} \sqrt{\sigma^2 + (\sin(\frac{\omega t}{2}))^2}} \right]$$

$$+ \frac{\omega a \frac{1}{\sqrt{2}}}{\sqrt{2} \sqrt{\sigma^2 + (\sin(\frac{\omega t}{2}))^2}}$$

(19)

Where ($a_1$) is the length of the crack reached at excess energy release.

Using a program and solved by Mathlab and using Newton Raphson method, from Eq. (19) it could be calculated the velocity of the crack growth $\frac{da}{dt}$ for each time required to reach the crack length $a_1$. Also the value of the limit value $\frac{da}{dt}$ for $a_1 > a_0$ is found to be less then unity.

Verification for the theoretical results

The principle stresses for the element of the non-proportional multi axial cycling loading can be calculated as [4]:

$$\sigma_{1.2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

(20)

For the case of this study the principal stresses can be written as:

$$\sigma_{1.2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

(21)

And the angle of the plane of principal stresses will be inclined with the horizontal axis can be represented as:

$$\tan2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

(22)

From Eq. (23) it can be shown that the angle of inclined of the principal plane with the horizontal axis will be various depending on the value of $\omega t$.

Since the orientation of the crack in the plate in the $y$ axis then it can be represented the crack is inclined with direction of the principal stresses by angle $\theta_p$. See Fig. 3.

Then the stress intensity factor for mode I and mode II can be written as:

If the value of $\sigma_2 = a\sigma_1$ then and $\sigma_2$ is a compressive stress always from Mohr circle for our study.

$$K_i = \frac{\sigma_1 \sqrt{\pi a}}{2} \left\{ (1 + \alpha) + (1 - \alpha) \cos 2\theta_p \right\}$$

(24)

$$K_{II} = \frac{\sigma_1 \sqrt{\pi a}}{2} \left\{ (1 - \alpha) \sin 2\theta_p \right\}$$

(25)

Then $\Delta K_i = K_{i\text{max}} - K_{i\text{min}}$ where $K_{i\text{max}}$ is depend on the value of $\alpha_1$ max when $\sin \omega t = 1$ and $\alpha_1\text{min}$ when $\sin \omega t = 0$

Also $\Delta K_{II}$ is depend on $\alpha_2\text{max}$ and $\alpha_2\text{min}$ then the mixed mode of I and II give [10]:

$$\Delta K_{eq} = \left\{ \Delta K_i^2 + \Delta K_{II}^2 \right\}^{0.5}$$

(26)

The crack growth can be calculated using Paris law as:

$$\frac{da}{dN} = C (\Delta K_{eq})^m$$

(27)

Two ways have been used in this research to solve Eq. (27)

1- From experiment $\frac{da}{dN}$ can be measured and the value of $C$ and $m$ can be obtained for composite nano material under multi axial loading from plotting $\log \frac{da}{dN}$ against $\log \Delta K_{eq}$ and the slope of the line m and the intersect of the line with the log $\frac{da}{dN}$ is the value of C. The value $da$ from Eq. (27) have been compared with da obtained from Eq. (19) by taking in the consideration the value of (N) number of revolutions per minute.
Table 1 – Mechanical properties.

<table>
<thead>
<tr>
<th>Type of material</th>
<th>Pure Cu</th>
<th>MWCNTs + Cu</th>
<th>SWCNTs + Cu</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength</td>
<td>68.73</td>
<td>114.25</td>
<td>184.58</td>
<td>Mpa</td>
</tr>
<tr>
<td>Compression Strength</td>
<td>305</td>
<td>763</td>
<td>1018</td>
<td>Mpa</td>
</tr>
<tr>
<td>Young Modulus</td>
<td>100</td>
<td>150</td>
<td>187.5</td>
<td>GPa</td>
</tr>
<tr>
<td>Micro Hardness (HV)</td>
<td>54</td>
<td>99</td>
<td>112</td>
<td>Kg/mm²</td>
</tr>
<tr>
<td>Kc (mode I)</td>
<td>3.358</td>
<td>6.678</td>
<td>8.352</td>
<td>Mpa·m¹²</td>
</tr>
<tr>
<td>Kc (mode II)</td>
<td>4.7699</td>
<td>4.49</td>
<td>4.9</td>
<td>Mpa·m¹²</td>
</tr>
<tr>
<td>Density</td>
<td>7.98</td>
<td>7.8</td>
<td>7.7</td>
<td>g/cm³</td>
</tr>
</tbody>
</table>

Table 2 – Paris Law’s constants.

<table>
<thead>
<tr>
<th>Pure Cu</th>
<th>MWCNTs + Cu</th>
<th>SWCNTs + Cu</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimentally Uni-axial</td>
<td>C</td>
<td>10⁻¹⁸</td>
<td>9*10⁻⁹</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>3.23</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>Multi-axial</td>
<td>C</td>
<td>10⁻¹³</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>2.68</td>
<td>1.42</td>
</tr>
<tr>
<td>Theoretically Uni-axial</td>
<td>C</td>
<td>10⁻¹⁵</td>
<td>6*10⁻⁸</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>3.1</td>
<td>1.103</td>
</tr>
<tr>
<td></td>
<td>Multi-axial</td>
<td>C</td>
<td>10⁻¹⁴</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>2.14</td>
<td>1.01</td>
</tr>
<tr>
<td>Differences Uni-axial</td>
<td>C</td>
<td>9*10⁻¹⁰</td>
<td>5*10⁻⁸</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>0.13</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>Multi-axial</td>
<td>C</td>
<td>9*10⁻¹⁰</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>0.54</td>
<td>0.41</td>
</tr>
</tbody>
</table>

2-In the multi axial cyclic loading applied in this research the values of the constants C and m are depend not only on the mechanical properties of the materials but its depends also on the ratio of r to σ, so the value of da/dn can be found from Eq. (17) by taking in the consideration the value of N from each time (t), and substitute in Eq. (27) to find the values of the constants c and m theoretically. Also for verification we can obtain the following formula to calculate (tₙₐ) theoretically (Eqs. (28–30) from the references [13-15] respectively).

\[ \theta_C = \pm \arccos \left( \frac{3K_n^2 + K_i}{K_i + 9K_n^2} \right) \]  
\[ \theta_C = \tan^{-1} \left( \frac{K_i}{4K_n} - \frac{1}{4K_n} \left( \frac{K_i}{K_n} \right)^2 + 8 \right) \quad K_i > 0 \]  
\[ \theta_C = \tan^{-1} \left( \frac{K_i}{4K_n} + \frac{1}{4K_n} \left( \frac{K_i}{K_n} \right)^2 + 8 \right) \quad K_i < 0 \]  

3. Experimental work

Experimental work is divided into three stages the first stage is to Produce plates from a Nano composite material (Cu + MWCNTs and Cu + SWCNTs), the second stage is to find the mechanical properties of the new materials, and the third stage is to design a new device for applying multi-axial cyclic loads.

Stage One: The best method for industrial work and with the best dispersion of CNTs in Cu powder is by using a high energy planetary ball milling device. The copper powder used with particle size ≤ 45 μm (325 mesh) with 99.99% purity with bulk density = 1.62 g/cm³. SWCNTs carboxyl groups (-COOH) purity ≥ 85%, with bulk density = 0.05 g/cm³, outer diameter 1.8 ± 0.4 nm length 5 μm) and MWCNTs carboxyl groups (COOH) purity > 97%, with bulk density = 0.18 g/cm³ outer diameter 20-40 nm length <10 μm). The high energy planetary ball milling device used with speed 300 r.p.m and with Steel balls coated by Tungsten (50 balls with 5 mm diameter and 7 grams for each ball) and the jar also made of steel and coated by tungsten (65 mm outer diameter, 50 mm inner diameter 100 mm length). It’s used to give perfect mixing and milling without notches. CNTs have been put in the jar then added Acetone 80 ml and mixed together then added the copper powder with the balls (the ratio of balls to composite powder is 3.5:1 (Cu 94 vol% and CNTs 6 vol%)) then close the jar tightly and run the device for 2.5 hours after that we add more Acetone 80 ml and run the ball milling for the other 2.5 hours. Every 5 hours 100 grams of the composite material will be mixed. After that, the sample was dried on the air to remove the Acetone for 2 hours at room temperature then put it in a closed package (in this research Acetone was used because it is available for continuous mass production of high strength yarns with a wide range of diameters, especially ultra-thin yarns) [16].

After that, the composite powder was pressed 600 Mpa in close steel die, with hydraulic pressing machine, then directly the sample was sintered in a tubular vacuum furnace till 900 C° for 120 minutes [17,18] (from 900°C takes 20 minutes) and the vacuum was 76 cm Hg (for vacuum used Quartz Glass tube with diameter 55 mm length 1000 mm), then turn off the furnace till the room temperature with a vacuum. For pressing the powder it was fabricated a new die to produce a flat plate with the dimensions (25, 150, 2 mm). That die was made off tool steel metal to hold out the 600 Mpa [18] stress (around 210 Ton pressing). Also was used a circular die to produce a sample for compression test with a diameter of 10 mm and height 50 mm, to obtain the mechanical properties.
Stage Two: The Mechanical properties was found by doing tensile test with a samples standard ASTM E8-04 (sub-size) specimens fabricated by using Wire-cut machine and for Com- pression test the sample was prepared with close cylindrical die with diameter 10 mm and length 50 mm and its produce the samples with diameter 10 mm and height 10 mm, Also Micro hardness Vickers was found by taking 10 tests for each sample then the average was taken, and the critical stress intensity factor Kc was found experimentally for Mode I and Mode II of fracture mechanics with crack length (2a =5 mm) (Mode III was neglected because it has a very small effect comparing with mode I and Mode II) see Table 1. Paris Law constant (c and m) calculated experimentally by using Dino Camera and finding Log (da/dN), Log (ΔK) for all three materials, also it’s calculated theoretically as shown in Table 2. Kc for mode I was found for our sample by applying a tensile load with very low speed 0.01 mm/sec and we fixed a Dino Camera at the time of crack starting to grow the critical stress was found then multiply it by Stress Safety factor (SSF) and by √πa, also the same for Kc Mode II but we fabricate a new grippers to apply the shear load. (See Fig. 4).

Stage Three: To design a new device for applying the multi-axial cyclic load it’s needed to know the three modes of fracture mechanics. Mode I is for tension loads (opening the crack) in a plane, Mode II is for shear loads also it’s in-plane loads and Mode III is for tier loads (torsion loads) out of the plane. This cause will consider Mode I and II (as a linear elastic fracture mechanics) plane stress. The tension load it will be constant (tension stress = 30 Mpa) and shear load will be cyclic load repeated (Max = 80 Mpa and Min = Zero). The new mechanism, contains a 3 phase motor (3000 r.p.m, 2 Hp ) with AC drive to control the speed of the motor and its input is Single Phase and its output power is three phases, the motor was connected to the crankshaft by a belt (V) type then the crank joined with slot link that connected to shear’s gripper that con-

Fig. 4 – Grippers For shear test for cracked sample

Fig. 5 – Top view of Multi-axial Cyclic load Machine
connected with slot slide from the down to make the movement on one direction only and connected to a dial gauge at its end, the other side gripper is fixed as shown in Fig. 6. For constant tension load was used a manual hydraulic pressing machine that presses the oil into a hydraulic jack, the piston of the jack is connected to the tension’s Gripper and the other gripper is fixed on the sliding base (all the mechanism of tension load are connected with the sliding base to separate the effects of a shear load from it) as shown in Fig. 5.

The constant tension was applied firstly (30 Mpa) after that the grippers of shear load was tighten, then turn on the motor from AC drive and fixing the speed of rotation at 1000 r.p.m (Tachometer was used to read the r.p.m from the crank shaft). The dial gauge was fixed on the end of shear grippers (using a slow motion camera video to record the reading of dial gauge it was 0.2 mm repeated). As the dimensions of our samples and G (modulus of rigidity of each material that was found from Modulus of Elasticity (E) and Poisson ratio (V), $G = E / (2^* (1 + V))$) assume the materials are isotropic as shown in SEM test Fig. 7, (Our method gave us good dispersion of CNTs in Cu). The shear stress was Max = 80Mpa and Min = zero, for pure copper. The crack was created at the One-third of our samples, and its dimension was 0.7 mm width, 5 mm length and depth 2 mm, it was used Wire-Cut machine to create it.

4. Results

In the results, two main points have been obtained from the theoretical, ABAQUS and experimental analysis is shown in Figs. 8–10 which represent the relation between crack lengths (a) against the Number of cycles (N). In the first point, its seen that (SWCNTs) plate have more than 100,000 cycles then the crack growth will be accelerated while (MWCNTs) have more than 80,000 and the pure copper have less than 70,000 cycles which have less arrest to crack growth.

The second point, its seen that the rate of crack growth is very high in (SWCNTs) when compared with (MWCNTs) and (Pure Copper) although that the life of compound cyclic loading has more than 160,000 cycles in the (SWCNTs) and the (MWCNTs) is 140,000 cycle and in the (Pure copper) is about 125,000 cycle. Fig. 11 it shows the final stage contour and the crack path for our sample, also its very close to the reality of our test as the crack growth on the right side more than the left. The angle of the crack growth (θc) for pure copper is around 27° experimentally, analytically and theoretically as mentioned in Eqs. (28,29, and 30), for MWCNTs & SWCNTs the angle of crack growth is around 35°, it can be used to verify the experimental work.
Table (2) shows Paris Law’s constants (c & m) were calculated experimentally and theoretically for multi-axial cyclic loading and uniaxial cyclic loading and it’s seen the effect of loading on its values. Also, the error percentage was found less than 20% for multi-axial cyclic loading, as mention above there are no efficient models or parameters that demonstrate multiaxial fatigue behavior [19]. Uniaxial cyclic loading has a very small error percentage (3%-4%), because of the theory gives the exact solution and experimental work gives the actual results. See Figs. 8–10.

5. Conclusion

Firstly, in this paper, we want to show that the easiest, fastest and more efficient way to produce a Nano composite material (CNTs+Cu), and it can be used in a wide range for industrials. The use of Acetone as a solvent is more effective than the other solvents and shows excellent results especially with CNTs-COOH. The mechanical properties for pure copper was improved in general around 1.8 to 3.7 times by adding CNTs-COOH. Also in this article it was seen the effect of CNTs on the crack speed and the fatigue life of multi-axial loading, at a chosen point in each material in the secondary stage of crack growth. Then comparing the results with ABAQUS program and finding the error percentage for each case (1.5% - 4%), it’s accepted for this case study because the numerical work is a finite element and it has some error depending on the mesh size and shape and other features. From the results it’s seen that the life (N) of SWCNTs+Cu is more than the others but its crack growth faster than MWCNT+Cu and Pure copper, so its indicated to say that SWCNTs+Cu is brittle.

6. Disclosure

The ABAQUS Program and MATLAB software were used in this study.
Conflicts of interest

The authors declare no conflicts of interest.

REFERENCES
