Original Article

A constitutive relation of AZ80 magnesium alloy during hot deformation based on Arrhenius and Johnson–Cook model

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ABSTRACT

In order to understand the constitutive behavior of as-cast AZ80 with large grain size, the uniaxial hot compression tests were carried out over a series of isothermal upsetting experiments. The maximum deformation degree was 65%. The experimental temperatures were 523 K, 573 K, 623 K and 673 K and the strain rate was 0.001 s⁻¹, 0.01 s⁻¹, 0.1 s⁻¹, and 1 s⁻¹. The stress–strain curves can be divided into three stages which are work hardening stage, softening stage, and steady-state stage at low strain rate and high temperature, while the steady-state stage cannot be observed at low forming temperature and high strain rate because of incomplete dynamic recrystallization. The Arrhenius type relation predicts the peak stress with high accuracy but cannot satisfy the strain relevant requirement. The Johnson–Cook model shows an inappropriate ability to describe the constitutive behavior in this case. Therefore, a new mathematical model (a segmented model) with high prediction accuracy based on the modified Arrhenius type relation (including strain rate) and Johnson–Cook model is proposed. The modified Arrhenius type relation is used to reflect the constitutive behavior before the peak strain and the modified Johnson–Cook model is aimed at showing the stages after peak strain.

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1. Introduction

Magnesium alloy, the lightest structural metallic material until now, has extensive application prospects in many fields especially in automotive and aircraft industrials because of its low density and high strength/weight ratio [1–5]. While low strength, poor formability and limited ductility have restricted its further application due to the hexagonal close packed (HCP) crystal structure which has limited slip system at low temperature [6]. Fortunately, the warm or hot working condition can activate the additional slip systems, for example, non-basal and (c+a) slip become sufficiently accessible by thermal activation to improve the workability of magnesium alloy [7,8]. During the hot forming process, magnesium alloy generally undergoes hardening effect (work hardening) and softening effect (dynamic recovery (DRV) and dynamic

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recrystallization (DRX) [9–12]. These behaviors will change following the changes of forming conditions such as temperature, strain rate and deformation degree. Thus, it is greatly important to understand the deformation behavior of Mg alloy for optimizing the forming process and reducing the rejection rate. Generally, the constitutive relationship including process parameters is utilized to represent the plastic flow behaviors during the deformation processes. It is the key and base of solid mechanics and most analytic calculations (e.g., the finite element method (FEM) simulation). It also can be used to analyze and predict the damage failure, strength and lifetime during the deforming process. Many mathematic models were exploited to character materials with different features such as Johnson–Cook (J–C) model [13] and Zerilli–Armstrong (Z–A) model [14,15]. The J–C model is proposed to describe the plastic flow behaviors with large deformation degree, high strain rate and temperature, and is widely used in engineering because of simple form and a few parameters. While the assumption of independence among strain, strain rate and temperature reduce its accuracy. The Z–A model which is based on kinetics of dislocation motion is usually employed in face-centered cubic and body-centered cubic materials. As for Mg alloys, the Arrhenius type relation [16–18] is customarily used to show the constitutive relationship

$$
\sigma = \frac{1}{a} \left\{ \left( \frac{j \exp \left( \frac{Q}{R} \right)}{A} \right)^{1/n} + \left[ \frac{j \exp \left( \frac{Q}{R} \right)}{A} \right]^{2/n} + 1 \right\}^{1/2},
$$

where A and n material parameters, Q the apparent activation energy, a the stress multiplier, R the universal gas constant, i the strain rate and T the absolute temperature. This model is established for the peak stress or steady-state flow stress because it neglects the influence of strain makes. However, only prediction of peak and steady-state stress cannot satisfy the request of real deformation process because of microstructure evolution. Besides, method of interpolation has to be introduced in FEM simulation to calculate the stress at other strain, reducing the simulation accuracy. To solve this problem, many researchers relative associate a, n, A and Q with strain to get the strain-dependence constitutive relation, while the complicated pattern, troublesome solving process and small prediction range limit its further application.

Therefore, a constitutive model with higher prediction accuracy and wider range relevant with strain, strain rate and temperature is requisite. On the other hand, the raw materials used in most of researches are taken from small ingot with small grain size, and few studies focus on the large billet with large grain. For instance, the largest grain size of AZ91 alloy for study of compression behaviors investigated by Cerri et al. was 30 μm [19]. The average grain size of AZ31B alloy sheet for constitutive modeling studied by Ulacia et al. was 10 μm [20]. The biggest grain size of AZ61 alloy used in constitutive analysis by Liao et al. was about 60 μm [21]. The equiaxial original material with about 12 μm grain size was reported by Cheng et al. for flow stress equation building [22]. Consequently, in the present work, one kind of constitutive relation that combined the modified Arrhenius type relation and J–C model was investigated to describe the plastic flow behaviors of AZ80 alloy with large original grain size. The true stress–strain curves were obtained by the hot compression at the strain rate of 0.001–1 s\(^{-1}\) and the temperature of 523–673 K.

2. Experimental procedure

The chemical composition (wt.%) of AZ80 alloy used in this work is given in Table 1. The hot compression procedure is shown in Fig. 1a. The sampling method and sample’s original macrostructure with the average grain size of 1400 μm are shown in Fig. 1b. The experimental samples were taken from a 9255 billet that was homogenized at 400°C for 12 h in order to remove second phase and developed dendrite structure. The compression specimens, with the diameter of 8 mm and the length of 12 mm, were machined with their cylinder axes that parallels to the axial line direction of the billet. These specimens were polished firstly and then were compressed at 523 K, 573 K, 623 K and 673 K on the Gleeble-3500D thermal simulated test machine with 65% maximum deformation degree. The strain rate (\(\dot{\epsilon}\)) was set as 0.001 s\(^{-1}\), 0.01 s\(^{-1}\), 0.1 s\(^{-1}\), and 1 s\(^{-1}\) respectively. Graphite sheets and lubricating oil were utilized to decrease the surface friction between specimen and indenter, promoting the accuracy of experimental results. As shown in Fig. 1a, prepared sample was heated up to stated temperature with the 5 K/s heating rate and then was held at settled temperature for 180 s in order to obtain the sample with uniform temperature. During the compression processing, the experimental data was gathered and recorded by the digital control system of the machine. After compression, the samples were quenched by cooled water instantly to maintain the evolutionary microstructure during the compression. The microstructures were investigated after the compression using optical microscopy (OM).

<table>
<thead>
<tr>
<th>Table 1 – Chemical composition of AZ80 alloy (wt.%).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alloy</td>
</tr>
<tr>
<td>AZ80</td>
</tr>
</tbody>
</table>

Fig. 1 – (a) Schematic illustrations of isothermal compression process. (b) Position and method of sampling for experimental specimen and macrostructure of original experimental material.
3. Results and discussion

3.1. Analysis of flow stress behavior and metallographic structure

The compressive true stress–strain curves of as-cast AZ80 magnesium alloy are demonstrated in Fig. 2a–d, which suggests a kind of constitutive behaviors strongly associated with temperature and strain rate. Comparing these curves with each other, it is observed that for a specific strain rate, the flow stress decreases markedly with temperature. The reasons behind this phenomenon are as follows. Firstly, the increase of temperature is able to enhance the effect of thermal activation energy resulting in increment of kinetic energy and decline of critical shear stress among atoms. Secondly, softening effect caused by DRV and DRX is improved as well with increase of temperature. Thirdly, some texture evolution may happen; for example, some lattice transform to plasticity favorable form. Fourthly, the intercrystalline shear resistance decreased significantly leading to easy intercrystalline slipage, and consequently, stress concentration due to inhomogeneous deformation between adjacent grains is released. On the other hand, for a fixed temperature, the flow stress generally increases following the increase of strain rate. With the enhancement of strain rate, the time of softening effect is reduced because of quicker deformation, leading to the increase of deformation resistance.

It is obvious that all the curves have obvious single-peak, suggesting that they belong to typical dynamic recrystallization type [16,23]. At micro-strain stage, because of the work hardening effect, the flow stress increases apace following the growth of strain with little DRV, suggesting that the hardening effect is much larger than the softening one. Then, with the increase of deformation degree, the dislocation density keeps increasing leading to enhancement of the softening effect (DRV or DXR), resulting in the decreased slope of stress–strain curve. Then the slope is continuously reduced to zero meaning that the strengthening effect is absolutely equal with softening effect, and the flow stress arrives at maximum, namely the peak stress. When stress surpasses the peak stress, the DRX softening effect is larger than the strengthening effect, leading to a strain softening tendency on the curve. Then, with further increase of deformation degree, especially at high temperature (623 K and 673 K) and low strain rate (0.001 s⁻¹ and 0.01 s⁻¹), the curves show a steady-state trend because of balance between softening effect (DRX) and hardening effect (grain refinement effect) [24]. While, it is noticed that continuous downward tendency are observed at low temperature and high strain rate (seen in Fig. 2a) without the steady-state stage, suggesting that the softening effect caused by DRX exceeds the hardening effect.

Compared with these strain–stress curves with other kind of alloys, same downward trend can also be seen in high entropy alloys [25], but the descent degree is higher than AZ80,
particularly in high strain rate [26]. However, in the cases of superalloys [27] and AA6063 aluminum alloy [28], very slight downward trends are seen after the peak strain, suggesting the steady-state stage after peak strain.

Analyzing microstructure after hot compression is a good way to understand the deformation mechanism. Dynamic recrystallization and twinning are two main modes during the hot process for magnesium alloy [29].

Fig. 3 shows the typical microstructure of AZ80 alloy after compression at the strain rate of 0.001 s\(^{-1}\) and 0.1 s\(^{-1}\) and at temperatures of 523 K, 573 K, 623 K and 673 K, respectively. When the strain rate is 0.001 s\(^{-1}\), samples get enough time for microstructure evolution. At the lower temperature of 523 K (Fig. 3a), large size coarse grains are still observed showing an abnormal strip form after compression. Only small parts of areas can see refined recrystallized grain distributed in grain boundaries. The incomplete dynamic recrystallization also explains the continuous downward trend of flow stress shown in Fig. 2a. Under condition of the low temperature and strain rate, dynamic recrystallization is a good way to release internal stress. Following increase of temperature (at 573 K (Fig. 3b)), it is obvious that the DRX region is remarkably enlarged because of activated non-basal slip system [30]. When the temperature is 623 K (Fig. 3c), the DRX degree is further improved showing a well homogenous microstructure, which suggests that the DRX become the main deformation mechanism. However, some fine recrystallized grains are grown up at high temperature (673 K (Fig. 3d)) and low strain rate (0.001 s\(^{-1}\)), because of sufficient time of growth and appropriate temperature.

Comparing with Fig. 3a and Fig. 3e, it can be seen that at 0.1 s\(^{-1}\) strain rate, there are a large number of twin crystals existed with few DRX region, suggesting that twinning is the main deformation mechanism [31]. At low temperature, Jain et al. had demonstrated that the slip system cannot be activated sufficiently [32], resulting in twinning dominate the plastic deformation. However, the deformation degree caused by twinning is very small, which cannot reach critical strain value of RDX for large size grain, leading to hard occurrence of DRX. When the temperature is 573 K (Fig. 3f), larger dynamic recrystallized region can be observed, and it is noticed that the DRX mainly appear at twin crystal. Fig. 3g shows an optimal microstructure with sufficient fine and equiaxial recrystallized grain indicating that at this condition, DRX is easily achieved, promoting the dynamic softening effect. Same as Fig. 3d, in Fig. 3h, plentiful fine grains obtained by DRX show a grown tendency, deducing that high deforming temperature makes the local grains grow up.

### 3.2. Comparative analysis of Arrhenius type relation and J–C model

It is known that the Arrhenius type relation is a reliable method to predict the peak stress with different temperature and strain rate [33]. For instance, Quan et al. used the Arrhenius type relation to build the constitutive equation of AZ80 alloy [34]. Besides, the activation energy Q of DRX, the threshold energy which has to exceed for occurrence of grain boundary or new surface for nucleation, can be obtained during the modeling process. The peak stress of different deformation conditions are given in **Table 2**.

<table>
<thead>
<tr>
<th>Strain rate</th>
<th>523 K</th>
<th>573 K</th>
<th>623 K</th>
<th>673 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>117</td>
<td>79.1</td>
<td>54.2</td>
<td>34.2</td>
</tr>
<tr>
<td>0.01</td>
<td>143.6</td>
<td>98.4</td>
<td>70.8</td>
<td>52.9</td>
</tr>
<tr>
<td>0.1</td>
<td>172.9</td>
<td>129.1</td>
<td>93.6</td>
<td>70.4</td>
</tr>
<tr>
<td>1</td>
<td>187.9</td>
<td>157</td>
<td>123</td>
<td>79</td>
</tr>
</tbody>
</table>

### Table 2 – Value of \(\sigma_p\) under different deformation conditions.

In order to obtain the parameters \(n, \alpha\) and \(Q\) in Eq. (1), some equations should be utilized as following

\[
\begin{align*}
\dot{\varepsilon} &= A F(\sigma) \exp\left(-\frac{Q}{RT}\right) \\
F(\sigma) &= \left\{ \begin{array}{ll} 
\sigma^n & \text{for } \sigma > 0 \\
\exp\left(\beta \sigma\right) & \text{for } \sigma = 0 \\
\left[\sinh\left(\alpha \sigma\right)\right]^n & \text{for } \sigma < 0 
\end{array} \right. 
\end{align*}
\]

(2)

Then, based on Eq. (2), the value of \(n, \beta\) and \(\alpha\) can be obtained by \(n = \partial \ln \dot{\varepsilon}/\partial \ln \sigma, \beta = \partial \ln \dot{\varepsilon}/\partial \sigma\) and \(\alpha = \beta/n\) after natural logarithms treatment of Eq. (2). After
calculation, \( n = 7.2248, \alpha = 0.01068 \) and \( Q = 190.35629 \text{kJ/mol} \) are obtained. Regarding parameter \( A \), another parameter called Zener–Hollomon parameter, \( Z \), should be introduced. \( Z \) parameter describes the effects of temperature and strain rate on deformation behaviors. The relationship between peak stress and \( Z \) can be seen in Eq. (3), and finally the \( A = 3.52904 \times 10^{14} \) is obtained. After gathering all the parameters, the peak stress can be obtained, as shown in Eq. (4)

\[
\ln Z = n \ln \sinh (\alpha \sigma_p) + \ln A \tag{3}
\]

\[
\sigma_p = 93.66365 \ln \left\{ \left( \frac{\dot{\epsilon} \exp \left( \frac{190.35629}{8157.677} \right)}{3.52905 \times 10^{14}} \right)^{1/2} + 1 \right\}^{1/2} \tag{4}
\]

The relative error (RE), average absolute relative error (AARE) and multiple coefficient of determination (R) computed by Eq. (5) are used to value the accuracy between the experimental data and fitting results.

\[
\begin{align*}
RE &= \frac{Q_i - q_i}{Q_i} \times 100% \\
AARE &= \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Q_i - q_i}{Q_i} \right| \times 100% \\
R &= \sqrt{1 - \frac{\sum_{i=1}^{n} (Q_i - q_i)^2}{\sum_{i=1}^{n} (Q_i - \bar{Q})^2}} \tag{5}
\end{align*}
\]

where \( Q_i \) is the experimental data, \( q_i \) is the value calculated by equation, \( \bar{Q} \) is the average value of the experimental data and \( n \) is the number of data.

Fig. 4 shows the correlation between experimental and calculated peak stress using Eq. (4). The results reflect that almost all points locate in the region where the relative error ranges from –10% to 10%. Only two values that represent the deformation conditions of \( \dot{\epsilon} = 0.001 \text{s}^{-1}, T = 673 \text{K} \) and \( \dot{\epsilon} = 1 \text{s}^{-1}, T = 673 \text{K} \) are out this range. The AARE is 3.53%. These results also confirm that the Arrhenius type relation really has excellent accuracy in terms of peak stress, but it is not necessary for strain relevant usage requirement.

Another commonly used constitutive relation to describe the deformation behaviors of magnesium alloys is J–C model. For example, Mirza et al. have proved that the modified J–C model is able to describe the constitutive relationship for a rare-earth containing magnesium alloy [35], and the standard J–C model is shown as

\[
\begin{align*}
\sigma &= (A + B \varepsilon^m) \left( 1 + C \ln \dot{\varepsilon} \right) (1 - T^{-m}) \\
T^* &= \frac{T - T_r}{T_m - T_r} \\
\dot{\varepsilon}^* &= \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \tag{6}
\end{align*}
\]

where \( A \) material constant (yield strength generally), \( T^* \) non-dimensional temperature, \( T_r \) reference temperature, \( T_m \) melting point (829 K), \( m \) temperature sensitivity coefficient, \( \dot{\varepsilon}^* \) non-dimensional strain rate, \( \dot{\varepsilon}_0 \) reference strain rate, \( C \) parameter relevant with strain rate and \( B \) material constant. The first term of J–C model reflects material characteristic under reference temperature and strain rate, and the second and third term reflects the temperature softening and strain rate hardening effects respectively. Generally, the quasi-static condition (a deformation condition at low strain rate and temperature) is regarded as reference temperature and strain rate. At reference condition, the model is changed as \( \sigma = A + B \varepsilon^n \), and then natural-logarithm treatment were implemented on both sides of equation, resulting in \( \ln (\sigma - A) = n \ln \varepsilon + \ln B \). Therefore, the value of slope and intercept in linear fitting of \( \ln (\sigma - A) \) versus \( \ln \varepsilon \) obtain \( n \) and \( B \). However, as shown in Fig. 5, in this case, there is worse linear dependencies between \( \ln (\sigma - A) \) and \( \ln \varepsilon \) in each deformation temperature form 523 K to 673 K at 0.001 s\(^{-1}\) strain rate. The reason behind this phenomenon is the contribution of DRX after peak stress, which agrees with the research reported by Abbasi-Bani [36].

Fig. 4 – Correlation between experimental and calculated peak stress.

Fig. 5 – Relationship between \( \ln (\sigma - A) \) and \( \ln \varepsilon \) at strain rate of 0.001 s\(^{-1}\) and temperature of 523–673 K.
3.3. A new model based on modified Arrhenius type relation and J–C model

According to the discussion above, there are positive slopes of stress–strain curves represented a strain-harden trend before peak strain and negative slopes represented the softening tendency after peak strain. Therefore, two stages need to be considered respectively, and consequently, it is greatly significant to define the peak strain accurately. Sellars gave notice to the relation between the peak strain $\varepsilon_p$ and grain size $d_0$ associated with Zener-Hollomon parameter, $Z$ [37]:

$$
\begin{align*}
\varepsilon_p &= A d_0^m Z^n \\
Z &= \dot{\varepsilon} \exp \left(\frac{Q}{RT}\right)
\end{align*}
$$

where $A$, $n$, $m$ material constant, $d_0$ original grain size, $\dot{\varepsilon}$ strain rate, $Q$ activation energy, $R$ universal gas constant and $T$ temperature. Because of some sampling point and homogenization treatment before experiment, the original grain size can be considered as equal. Then Eq. (7) can be simplified as

$$
\varepsilon_p = A \dot{\varepsilon}^b \exp \left(CT\right)
$$

In order to obtain the value of $A$, $b$ and $C$, natural-logarithm treatment is carried out on both sides of equation, resulting in $\ln \varepsilon_p = \ln A + b \ln \dot{\varepsilon} + CT$, and the average value of slope in linear fitting of $\ln \dot{\varepsilon}$ versus $\ln \dot{\varepsilon}$ (shown in Fig. 6a) and $T$ (shown in Fig. 6b) obtain $b = 0.0113$ and $C = -0.0049$, respectively. Then, the value of intercept in linear fitting of $\ln \varepsilon_p$ versus $\ln \dot{\varepsilon} + CT/b$ get $\ln A = 1.11896$ and finally obtain $A = 3.0617$. Consequently, the peak strain model is as

$$
\varepsilon_p = 3.0617^{0.0113} \exp \left(-0.00417\right)
$$

Fig. 7 shows the correlation between the experimental peak strain and calculated value using Eq. (9). It is evident that this kind of model is able to fit the peak strain accurately, because all points locate in the region where the relative error ranges from $-10\%$ to $10\%$ and the value of AARE is $4.7675\%$.

Considering the bullish tendency of stress-strain curves before peak strain because of work-hardening effect (shown in Fig. 2), an exponential type function is utilized to describe the constitutive behaviors of as-cast AZ80 alloy before peak strain:

$$
\sigma = A \left(1 - \exp \left(B \varepsilon\right)\right)
$$

Using nonlinear curve fitting can obtain the value of $A$ and $B$ under different deformation conditions, and the results are shown in Table 3. Fig. 8 shows the relation between $A$ and peak stress, reflecting a closely correlation with 0.9971 $R$ value and 4.0672% AARE value. Therefore, $A$ could be defined as peak

<table>
<thead>
<tr>
<th>Temperature</th>
<th>0.001s$^{-1}$</th>
<th>0.01s$^{-1}$</th>
<th>0.1s$^{-1}$</th>
<th>1s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 523 K</td>
<td>118.06544</td>
<td>144.48244</td>
<td>177.80516</td>
<td>191.89378</td>
</tr>
<tr>
<td>573 K</td>
<td>78.58164</td>
<td>97.3195</td>
<td>129.37991</td>
<td>160.88398</td>
</tr>
<tr>
<td>623 K</td>
<td>49.55807</td>
<td>69.32185</td>
<td>93.97152</td>
<td>125.25113</td>
</tr>
<tr>
<td>673 K</td>
<td>31.12617</td>
<td>50.36664</td>
<td>70.12818</td>
<td>94.54231</td>
</tr>
<tr>
<td>B 523 K</td>
<td>25.6999</td>
<td>21.06089</td>
<td>15.66625</td>
<td>17.24217</td>
</tr>
<tr>
<td>573 K</td>
<td>44.49303</td>
<td>55.61846</td>
<td>24.96629</td>
<td>19.68338</td>
</tr>
<tr>
<td>623 K</td>
<td>68.52912</td>
<td>53.74832</td>
<td>36.38243</td>
<td>22.82482</td>
</tr>
<tr>
<td>673 K</td>
<td>101.58451</td>
<td>86.751</td>
<td>46.53328</td>
<td>31.54839</td>
</tr>
</tbody>
</table>
stability ($\sigma_p$). Unfortunately, there is not a certain parameter relevant with B, meaning that further analysis is needed.

Firstly, the relation between B and strain rate and the relation between B and temperature are investigated, as shown in Fig. 9. There are strong linear correlations between $\ln \dot{\varepsilon}$ and $\ln B$, and between temperature and $\ln B$. Based on these relationships, B can be defined as a model associated with strain rate and temperature:

$$B = A_1 \dot{\varepsilon}^{B_1} \exp (C_1 T) \quad (11)$$

Then, the average value of slope in linear fitting of $\ln B$ versus $\ln \dot{\varepsilon}$ (shown in Fig. 9) and T (shown in Fig. 9b) obtain $B_1 = -0.13133$ and $C_1 = 0.00738$, respectively. To calculate the value of $A_1$, Eq. (11) can be transformed into another form, $\ln B = \ln A_1 + B_1 (\ln \dot{\varepsilon} + C_1 T/B_1)$. Then, the value of intercept in linear fitting of $\ln B$ versus $\ln \dot{\varepsilon} + C_1 T/B_1$ (shown in Fig. 10) get $\ln A_1 = -0.13133$ and finally obtain $A_1 = 0.26778$. Therefore, the B can be calculated by:

$$B = 0.26778 \dot{\varepsilon}^{-0.13133} \exp (0.00738T) \quad (12)$$

Therefore, the constitutive behaviors of as-cast AZ80 alloy before peak strain based on modified Arrhenius type relation can be shown as follows:

$$\sigma = \sigma_p \left( 1 - \exp \left( 0.26778 \dot{\varepsilon}^{-0.13133} \exp (0.00738T) \dot{\varepsilon} \right) \right)$$

$$\sigma_p = 93.66365 \ln \left( \frac{\dot{\varepsilon} \exp \left( \frac{190.35629}{8.1345T} \right) 1^{1/22478}}{3.52905 \times 10^{14}} \right)$$

$$+ \left( \frac{\dot{\varepsilon} \exp \left( \frac{190.35629}{8.1345T} \right) 2/22478}{3.52905 \times 10^{14}} \right) + 1^{0.5} \quad (13)$$

The stress–strain curves show downward tendencies after peak strain, as shown in Fig. 2a. It is also noting that similar forms are observed at different strain rate and temperature. Consequently, the modified J-C model is considered to describe the constitutive behaviors at this stage:

$$\sigma = (A_2 + B_2 \dot{\varepsilon} + C_2 \dot{\varepsilon}^2) \left( 1 + D_2 \ln \dot{\varepsilon}^* \right) \exp \left( \frac{E_2 + F_2 \ln \dot{\varepsilon}^*}{T - T_{ref}} \right) \quad (14)$$

where $T_{ref}$ reference temperature and $A_2$, $B_2$, $C_2$, $D_2$, $E_2$ and $F_2$ parameter associated with material. The first term of traditional J-C model is replaced by quadratic polynomial due to parabolic form of stress–strain curves after peak strain, and the third term is combined with strain rate in order to overcome the defect of no connection among third terms in traditional model, improving its accuracy.

In this study, the deformation condition of 523K and 0.001 s$^{-1}$ is regarded as the conference temperature and strain rate respectively, for purpose of calculating the
material constants. Then, at the conference deformation condition, Eq. (14) can be transformed as:

\[ \sigma = (A_2 + B_2 \varepsilon + C_2 \varepsilon^2) \]  

(15)

Substituting the experimental stress and strain data into Eq. (15), the relation between \( \sigma \) and \( \varepsilon \) is obtained using nonlinear fitting, as shown in Fig. 11, and consequently, the values of \( A_2 = 138.5314, B_2 = -103.4680, \) and \( C_2 = 32.9570, \) can be gotten from the fitting curve.

When the deformation temperature is 523 K, Eq. (14) can be converted as:

\[ \sigma = (A_2 + B_2 \varepsilon + C_2 \varepsilon^2) = 1 + D_2 \ln \dot{\varepsilon}^* \]  

(16)

Fig. 12 shows the relationship between \( \sigma/(A_2 + B_2 \varepsilon + C_2 \varepsilon^2) \) and \( \ln \dot{\varepsilon}^* \), and then the value of slope in linear fitting of \( \sigma/(A_2 + B_2 \varepsilon + C_2 \varepsilon^2) \) versus \( \ln \dot{\varepsilon} \) obtains \( D_2 = 0.1187 \).

In order to get the value of \( E_2 \) and \( F_2 \), a new parameter \( G \) is introduced, and make \( G = E_2 + F_2 \ln \dot{\varepsilon}^* \). Consequently, when the strain rate is 0.001 s\(^{-1}\), Eq. (14) can be written as follow:

\[ \ln \left( \frac{\sigma}{A_2 + B_2 \varepsilon + C_2 \varepsilon^2} \right) = G (T - T_{ref}) \]  

(17)

Fig. 13a shows the relationship between \( \ln \left[ \sigma/(A_2 + B_2 \varepsilon + C_2 \varepsilon^2) \right] \) and \( T - T_{ref} \). Hence, the value of \( G = -0.0097 \) could be obtained from the fitted line of \( \ln \left[ \sigma/(A_2 + B_2 \varepsilon + C_2 \varepsilon^2) \right] \) and \( T - T_{ref} \) as slope. Because of linear definition between \( G \) and \( \ln \dot{\varepsilon}^* \), the value of slope and intercept in linear fitting of \( G \) versus \( \ln \dot{\varepsilon}^* \) (shown in Fig. 13b) obtain \( E_2 = 0.000406 \) and \( F_2 = -0.00933 \), respectively.

Therefore, the constitutive behaviors of as-cast AZ80 alloy after peak strain based on modified J-C model can be described as follow:

\[ \sigma = \left( 138.5314 - 103.4680 \varepsilon + 32.9570 \varepsilon^2 \right) \left( 1 + 0.1187 \ln \dot{\varepsilon}^* \right) \left( \exp \left( 0.000406 - 0.00933 \ln \dot{\varepsilon}^* \right) \left( T - T_{ref} \right) \right) \]  

(18)

Finally, the constitutive equation of as-cast AZ80 alloy is as

\[
\begin{align*}
\sigma &= \sigma_p \left( 1 - \exp \left( -0.6093 \dot{\varepsilon}^* \right) \right) \dot{\varepsilon} \leq \dot{\varepsilon}_p \\
\sigma &= \left( 138.5314 - 103.4680 \varepsilon + 32.9570 \varepsilon^2 \right) \left( 1 + 0.1187 \ln \dot{\varepsilon}^* \right) \left( \exp \left( 0.000406 - 0.00933 \ln \dot{\varepsilon}^* \right) \left( T - T_{ref} \right) \right) \dot{\varepsilon} > \dot{\varepsilon}_p \\
\sigma_p &= 93.6636 \ln \left\{ \dot{\varepsilon} \exp \left( \frac{190.35629}{3.52905 \times 10^{14}} \right) \right\}^{1/7} 22478 \\
&\quad + \left[ \left( \dot{\varepsilon} \exp \left( \frac{190.35629}{3.52905 \times 10^{14}} \right) \right) \right]^{1/14} + 0.5 \\
\dot{\varepsilon}_p &= 3.0617 \dot{\varepsilon}^{0.0113} \exp \left( -0.0041 T \right)
\end{align*}
\]  

(19)

Fig. 14 shows the comparison between calculated (using Eq. (19)) and measured stress-strain curves of as-cast AZ80 alloy at different deformation conditions. It can be seen that the predicted flow stress agree well with the experimental values except a few areas. In order to further evaluate the accuracy of this new model, serial strain values of 0.05, 0.1, 0.15...0.9 were
selected to verify the correlation between computed (using Eq. (19)) and measured data of flow stress. As shown in Fig. 15, most of points locate in the RE range of −10% to 10%, and small AARE (4.8041%) and great correlation coefficient (0.9876) can be seen. Moreover, the frequency distribution plot showing a Gaussian distribution also confirms that this new model has decent prediction precision.

4. Conclusion

In this work, the hot compression tests were utilized in the as-cast AZ80 alloy with large grain size under a wide range of strain rate (0.001–1 s⁻¹) and temperature (523–673 K) to investigate its constitutive behaviors. The constitutive behaviors were analyzed by the combination of microstructure and true stress–strain curve, and a new constitutive model based on Arrhenius type relation and J–C model was put forward. The concrete results are as follows:

1. The true stress–strain curves can be divided to three stages that are hardening stage, softening stage and steady-state stage at high test temperature and low strain rate, while the steady-state stage cannot be observed under low temperature and high strain rate. The metallographic structures observation is consistent with the stress–strain curves showing that the twinning dominate the deformation with little DRX at low temperature and high strain rate, and on the contrary, full DRX can be obtained at high temperature and low strain rate.

2. The Arrhenius type relation can be used to calculate the peak stress with high accuracy while it cannot satisfy the strain relevant usage requirement. The J–C model shows a worse conformance in this case because of the dramatic softening tendency after peak stress.

3. Two models are utilized to describe the constitutive relation before and after peak strain, and the peak strain

![Fig. 14](image-url)  
**Fig. 14** – Comparison between calculated (using Eq. (19)) and measured stress–strain curves of as-cast AZ80 alloy at a strain rate of (a) 0.001 s⁻¹, (b) 0.01 s⁻¹, (c) 0.1 s⁻¹, and (d) 1 s⁻¹.

![Fig. 15](image-url)  
**Fig. 15** – Correlation evaluation between computed (using Eq. (19)) and measured data of flow stress.
can be calculated by $\varepsilon_p = A_1\varepsilon^B \exp(CT)$ well. The modified Arrhenius type relation $\varepsilon = \sigma_p (1 - \exp ((A_1 \varepsilon^B \exp (C_1 T)) \varepsilon))$ is used to reflect the reformation behavior before peak strain, and the modified J-C model $\sigma = (A_2 + B_2 \varepsilon + C_2 \varepsilon^2) (1 + D_2 \ln \varepsilon^\cdot) (\exp ((E_2 + F_2 \ln \varepsilon^\cdot) (T - T_\text{ref})))$ is used to describe the constitutive relation after peak strain. Consequently, a new constitutive relation that combines these two models is able to predict the constitutive behavior of as-cast AZ80 alloy with a high accuracy.

**Conflicts of interest**

The authors declare no conflicts of interest.

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